Reference Sheet

Some Limits: For any positive real number $n > 0$ we have:

$$\lim_{x\to\infty} \frac{1}{x^n} = 0 \qquad \lim_{x\to 0^+} \frac{1}{x^n} = \infty \qquad \lim_{x\to -\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} x^n e^{-x} = 0 \qquad \lim_{x\to\infty} \frac{e^x}{x^n} = \infty \qquad \lim_{x\to 0^+} x^n \ln x = 0$$

Some Differentiation Rules: Let f and g be differentiable functions.

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x) \qquad \frac{d}{dx}(f(x)\cdot g(x)) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2} \qquad \frac{d}{dx}(g(f(x))) = g'(f(x))f'(x)$$

Let a and n be real numbers with $a > 0$ and $a \neq 1$

$$\frac{d}{dx}(x^n) = nx^{x-1} \qquad \frac{d}{dx}(a^x) = a^x \ln a \qquad \frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a} \qquad \frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Some Trigonometric Identities:

$$\cos(-x) = \cos x \qquad \sin(-x) = -\sin x \qquad \tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Tangent Line: The equation of the tangent line to a curve $y(x)$ (given explicitly or implicitly) at a point (x_0, y_0) is: $y - y_0 = y'(x_0)(x - x_0)$

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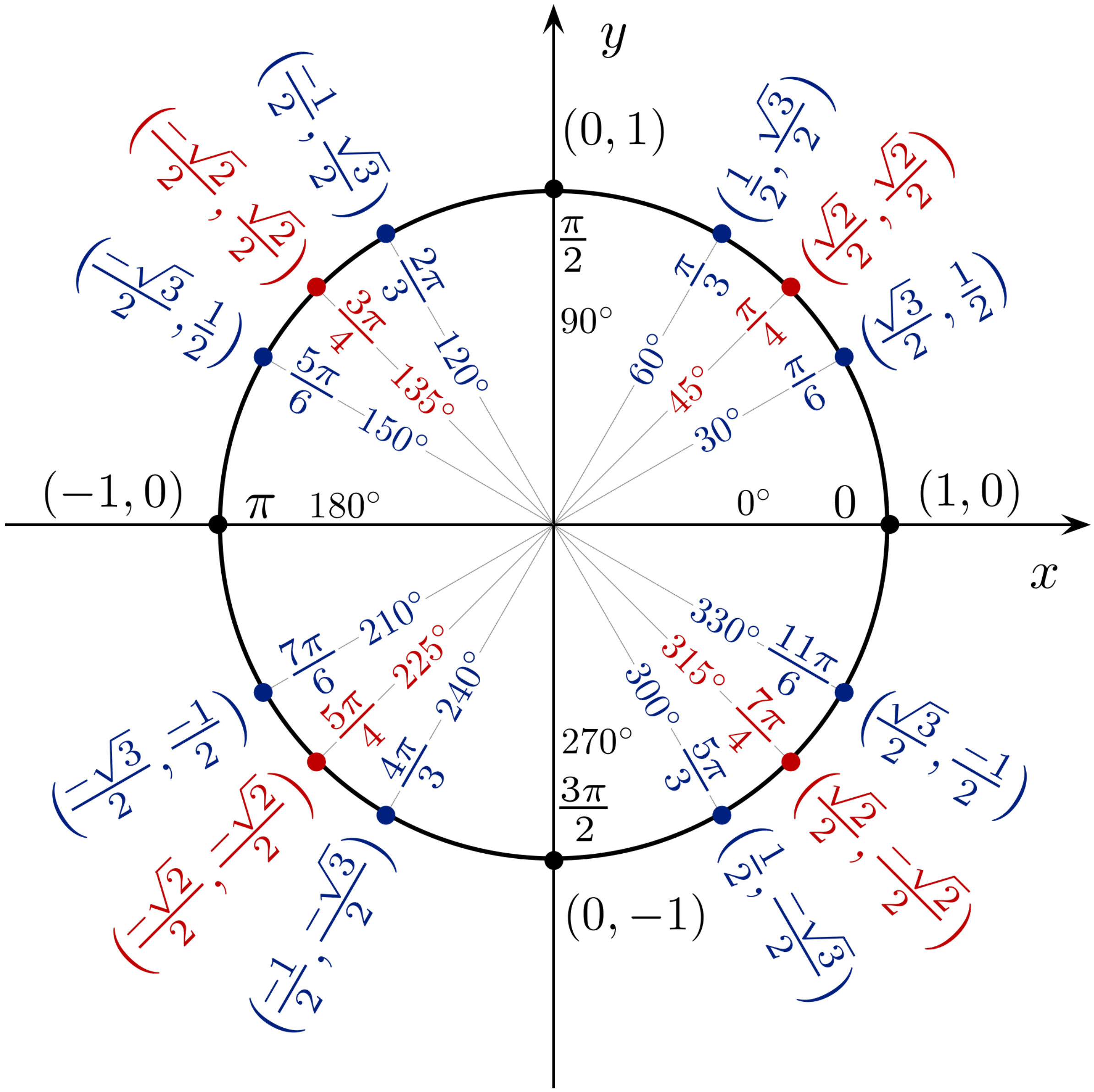

Example:

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \qquad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \qquad \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

$$\csc\left(\frac{\pi}{6}\right) = 2 \qquad \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \qquad \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$$

Reference Sheet

Some integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int a^{kx} dx = \frac{a^x}{k \ln a} + C,\ a \neq 1, k \neq 0$$

$$\int e^{kx} dx = \frac{e^{kx}}{k} + C,\ k \neq 0 \qquad \int x^{-1} dx = \int \frac{dx}{x} = \ln|x| + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C \qquad \int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C \qquad \int \csc x \cot x dx = -\csc x + C$$

$$\int \tan x dx = -\ln|\cos x| + C \qquad \int \cot x dx = \ln|\sin x| + C$$

Fundamental Theorem of Calculus:

$$\int_a^b f(x)dx = F(b) - F(a),$$

where F is any antiderivative of f. Consequently,

$$\frac{d}{dx}\int_a^x f(x)dx = f(x)$$

Some typical substitution cases: In the following indefinite integrals, the substitution $u = f(x)$ leads to a simple answer.

Form of the integral	**Result**
$\int [f(x)]^n f'(x)dx,\ n \neq -1$	$\int u^n du = \frac{u^{n+1}}{n+1} + C = \frac{[f(x)]^{n+1}}{n+1} + C$
$\int \frac{f'(x)}{f(x)} dx$	$\int \frac{du}{u} = \ln|u| + C = \ln|f(x)| + C$
$\int e^{f(x)} f'(x)dx,$	$\int e^u du = e^u + C = e^{f(x)} + C$

Integration by Parts:

$$\int u dv = uv - \int v du$$

Types of Differential Equations:

Elementary: $\frac{dy}{dx} = f(x)$; Separable: $\frac{dy}{dx} = \frac{p(x)}{q(y)}$ Linear: $\frac{dy}{dx} + p(x)y = q(x)$