9. Let $f(x) = \dfrac{1}{x^2 - 1}$. You may use that $f' = \frac{-2x}{(x^2-1)^2}$ and $f'' = \frac{6x^2+2}{(x^2-1)^3}$.

(a) (2 points) Find the vertical asymptotes of the graph of f.

(b) (2 points) Find the horizontal asymptotes of the graph.

(c) (2 points) Find the intervals where f is increasing.

Recall that $f' = \frac{-2x}{(x^2-1)^2}$ and $f'' = \frac{6x^2+2}{(x^2-1)^3}$

(d) (2 points) Find the intervals where f is concave up.

(e) (2 points) Find the maximal value of f in the interval $[4, 6]$

(f) (2 points) Sketch of the graph of $y = f(x)$.

9. Let $f(x) = \dfrac{1}{x^2 - 1}$. You may use that $f' = \frac{-2x}{(x^2-1)^2}$ and $f'' = \frac{6x^2+2}{(x^2-1)^3}$.

(a) (2 points) Find the vertical asymptotes of the graph of f.

$$f(x) = \frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} \Rightarrow \text{V.A are } \boxed{x=1,\ x=-1}$$

(b) (2 points) Find the horizontal asymptotes of the graph.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{x^2-1} = 0 \qquad \lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1}{x^2-1} = 0$$

so H.A. is $\boxed{y=0}$

(c) (2 points) Find the intervals where f is increasing.

increasing when $f'>0$ $\qquad f'(x)=0 \Rightarrow x=0$

$f'(x)$ undef. when $(x^2-1)^2=0 \Rightarrow (x-1)^2(x+1)^2=0 \Rightarrow x=\pm 1$

f': $+$ | -1 | $+$ | 0 | $-$ | 1 | $-$ $\Rightarrow \boxed{(-\infty,-1),(-1,0)}$

Recall that $f' = \frac{-2x}{(x^2-1)^2}$ and $f'' = \frac{6x^2+2}{(x^2-1)^3}$

(d) (2 points) Find the intervals where f is concave up.

concave up when $f''>0$

$f''(x)=0$ never, $f''(x)$ undef when $(x^2-1)^3=0 \Rightarrow (x-1)^3(x+1)^3=0 \Rightarrow x=\pm 1$

f'': $+$ | -1 | $-$ | 1 | $+$ $\Rightarrow \boxed{(-\infty,-1),(1,\infty)}$

(e) (2 points) Find the maximal value of f in the interval $[4, 6]$

cr # = 0 (outside int.)

$f(4)=\frac{1}{16-1}=\frac{1}{15}$ $\quad f(6)=\frac{1}{36-1}=\frac{1}{35}$

max is $\boxed{1/15}$

(f) (2 points) Sketch of the graph of $y = f(x)$.

f': $+$ | -1 | $+$ | 0 | $-$ | 1 | $-$

f'': $+$ | -1 | $-$ | 1 | $+$

(0,−1) rel max