$Midterm \ 1 - Math \ 241$

Friday, June 28, 2019

This is a closed-book exam. No calculators allowed. Justify your answers to obtain full credit (and partial credit, too). You have 75 minutes. This exam consists of 6 questions. Please verify that you have all pages. If you need scratch paper, please ask.

Name:_____

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(1) Answer the following questions for the function f graphed below:



(a) On what interval(s) is the function continuous?

(b) For which value(s) of a does $\lim_{x \to a} f(x)$ not exist?

(c) For what value(s) x is f(x) not continuous. Explain why.

(d) State the equation(s) of any vertical asymptotes.

(2) Evaluate the following limits. If the limit is ∞ or $-\infty$, please indicate as appropriate.

(a)
$$\lim_{x \to -2} \frac{x^2 + 5x + 6}{x^2 - 4}$$

(b)
$$\lim_{x \to \infty} \frac{4x^3 - 21x^2 + 3x}{x^2 + 13x - 7}$$

(c)
$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$

(d)
$$\lim_{x \to 0} \frac{2x}{\sin 3x}$$

(e)
$$\lim_{x \to \infty} \frac{\cos(x^2)}{x^4}$$

(3) Use the limit definition of a derivative to differentiate $f(x) = x^2 + 4$.

(4) Find the derivatives of the following functions. You do not need to simplify your answer.

(a)
$$f(x) = 3x^2 + \frac{1}{x^2} - \sqrt{\pi}$$

(b)
$$f(x) = \frac{8 + \sqrt[3]{x}}{3x - 5}$$

(c)
$$f(x) = \sin x + x \tan x$$

(d)
$$f(x) = (x^2 + 2x + 4)^{1/2}$$

(e)
$$f(x) = \sin^2(x^2 + 1)$$

(5) Show that $4x^3 - 6x^2 - 6x + 5 = 0$ has at least one solution. Be sure to state any theorems you are using and why you can use them.

(6) Find an equation of the line tangent to $3x^2 + 2xy + y = 0$ at (1, -1).