Math 241 Midterm 1 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, quizzes, and homework problems.

Previous Semester's Exam Problems

(1) Differentiate the following functions. You do not need to simplify your answers.

- (a) $y = 3x \sin x$
- (b) $y = \tan(x^2 + 1)$
- (c) $y = \dfrac{x+1}{x^2+2}$
- (d) $f(x) = 12x^2 - \dfrac{5}{\sqrt{x}} + 78$
- (e) $y = \dfrac{\sin^2 x}{2x^3 + 4x^2 + 7}$

(2) Find an equation of the tangent line to $x^2 + xy + y^2 = 3$ at $(1, 1)$

(3) Evaluate the following limits Be as specific as possible (i.e write ∞ or $-\infty$ instead of DNE when applicable):

- (a) $\displaystyle\lim_{x\to -1} \frac{x^2 - 5x - 6}{x+1}$
- (b) $\displaystyle\lim_{x\to 3^-} \frac{4}{(x-3)^2}$
- (c) $\displaystyle\lim_{x\to \infty} \frac{x^3 + x^2 + 2}{7x^3 + x + 1}$
- (d) $\displaystyle\lim_{x\to 25} \frac{x - 25}{\sqrt{x} - 5}$
- (e) $\displaystyle\lim_{x\to 0} \frac{\sin 4x}{5x}$

(4) Consider the function $f(x)$ given below. Find

(i) $\lim_{x \to k^-} f(x)$

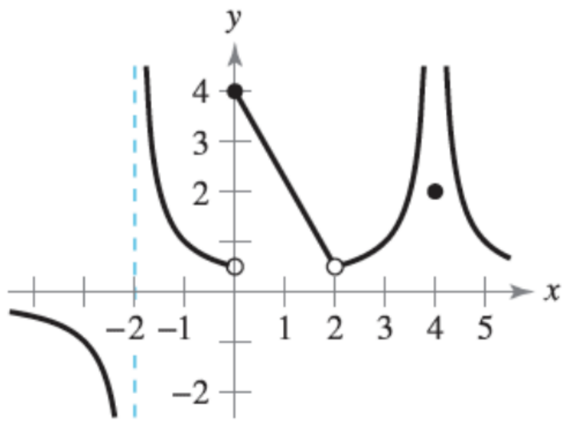
(ii) $\lim_{x \to k^+} f(x)$

(iii) $\lim_{x \to k} f(x)$

(iv) $f(k)$

(v) Is $f(x)$ continuous at k? (yes or no)

for each of the given values of k. If the given value does not exist, write "DNE", ∞, $-\infty$, or "undefined" as necessary:



(a) $k = -1$

(b) $k = 0$

(c) $k = 2$

(d) $k = 4$

(5) (a) Using the limit definition of a derivative, differentiate the following:

$$f(x) = x^2 - 3x - 1$$

(b) Find the equation of the line tangent to $f(x)$ at $x = 1$

(6) On which interval(s) is the following function continuous. Justify your answers using limits:

$$f(x) = \begin{cases} 1 - x^2 & x < -1 \\ 1 + x & -1 \leq x \leq 1 \\ -3 & x > 1 \end{cases}$$

(7) Evaluate $\lim_{x \to 0} (x^2 \sin(4x) + 1)$

(8) Show that the equation $x^3 - x^2 + 2x - 7 = 0$ has a solution in the interval $[1, 2]$. State any theorems you use to support your answer.

(9) Calculate the following derivatives:

(a)

$$y = \frac{x^3 + 1}{2 - x}, \qquad \frac{dy}{dx} =$$

(b)

$$h(t) = \cos^2(\pi t) + 3, \qquad h'(t) =$$

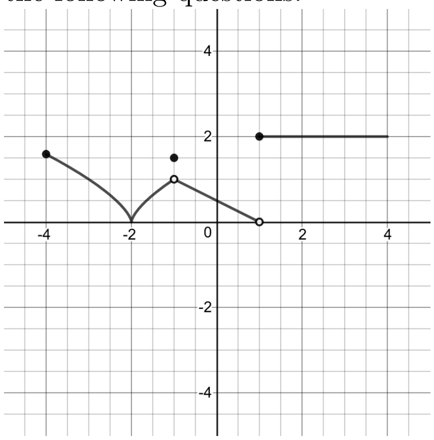
(c)

$$f(x) = \sin(x)(x^2 + 3) + x^{4/3}, \qquad f'(x) =$$

(d)

$$y^4 - \sqrt{x} + 2y = 2, \qquad y' \text{ at } (1,1)$$

(10) The function $f(x)$ is defined for $-4 \le x \le 4$ and is graphed below. Use the graph to answer the following questions:



(a) What is $\lim\limits_{x\to -1} f(x)$?

(b) What is $\lim\limits_{x\to 1} f(x)$?

(c) Give the intervals where $f(x)$ is continuous, be careful to include the endpoints if necessary.

(d) Does the function appear to be differentiable at $x = -2$? Explain why or why not.

(11) Let $f(x) = 5 + x - x^4$. Use the intermediate value theorem to show that there is at least one point where $f(x) = 0$.

Extra Practice Problems

(1) Calculate the following limits.

(a) $\lim_{t\to 1} \frac{t^2+t-2}{t^2-1}$

(b) $\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2}$

(c) $\lim_{x\to\infty} \frac{7x^3}{x^3-3x^2+6x}$

(d) $\lim_{x\to 0} \frac{1}{x^3-1}+1$

(e) $\lim_{x\to 1} \frac{3x-4}{x^2+x+1}$

(f) $\lim_{x\to -1} \frac{x^2+x-2}{x^3+1}$

(g) $\lim_{x\to -1^-} \frac{|x+1|}{x+1}$

(h) $\lim_{x\to 1} \frac{x^2-1}{x-1}$

(i) $\lim_{x\to\infty} \frac{2x^3-4x^2+5x}{17x^2+1}$

(j) $\lim_{x\to -\infty} \frac{3x^2+4}{x+7}$

(k) $\lim_{t\to 2^-} \frac{t+2}{t-2}$

(l) $\lim_{x\to 0} \frac{\sin x}{3x}$

(m) $\lim_{x\to 0} x\cos\left(\frac{1}{x}\right)$

(2) Describe on which intervals the following functions are continuous (show your work):

(a) $y = \frac{\sin x}{x-2}$

(b) $f(x) = \begin{cases} 3-x, & x<2 \\ \frac{x}{2}+1, & 2\le x<4 \\ 3, & x\ge 4 \end{cases}$

(c) $f(x) = \begin{cases} 1-x^2 & x<-1 \\ 1+x & -1\le x\le 1 \\ -3 & x>1 \end{cases}$

(d) Calculate the following limits. State clearly any theorems that you use.

i.

$$\lim_{x\to 3^-} \frac{|x-3|}{(6-2x)}$$

ii.

$$\lim_{t\to 0} \frac{\sin(3t)}{t}$$

iii.

$$\lim_{h\to 1} \frac{\sqrt{h+8}-3}{1-h}$$

iv.

$$\lim_{x\to 0} x^2\cos\left(\frac{3}{x}\right)-11$$

(e) i. **Use the definition of derivative** to show that the derivative of $f(x) = x^2 - x$ at $x = -2$ is -5, i.e. $f'(-2) = -5$.

ii. Find an equation for the tangent line to $f(x) = x^2 - x$ at $x = -2$.

(f) Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval $[-4, 4]$

(g) Use the definition of the derivative (limit definition) to find the derivatives of the following:

i. $f(x) = \sqrt{x}$

ii. $f(x) = x^2 - x$

iii. $f(x) = \frac{1}{x}$

(h) Consider the function $f(x) = 5 - x^2$.

i. Find the equation for the secant line to the graph of $f(x)$ that passes through the points $(1,4)$ and $(2,1)$.

ii. Find $f'(x)$ using the definition of a derivative.

iii. Find the equation for the tangent line to the graph of $f(x)$ at the point $(1,4)$.

iv. Find the equation for the tangent line to the graph of $f(x)$ at the point $(2,1)$.

(i) A particle is moving along the x-axis. Its position at time t is given by the function $s(t) = -2t^2 + 5t - 2$.

i. Find the particle's average velocity v_{av} between $t = 1$ and $t = 4$.

ii. Find the particle's instantaneous velocity at $t = 1$.

(j) Find the first derivatives of the following:

i. $y = 6x^2 - 10x - 5x^{-2}$

ii. $y = x^2 \sin x + 2x \cos x - 2 \sin x$

iii. $h(x) = x \tan\left(2\sqrt{x}\right) + 7$

iv. $y = \dfrac{\cot x}{1 + \cot\left(x^2 + x\right)}$

v. $y = \left(1 - \dfrac{x}{7}\right)^{-7}$

(k) Find equations for the tangent and normal lines to $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at $(-1,0)$.