Midterm 2 – Math 241

Friday, July 10, 2019

This is a closed-book exam. No calculators allowed.
Justify your answers to obtain full credit (and partial credit, too).
You have 75 minutes.
This exam consists of 5 questions.
Please verify that you have all pages.
If you need scratch paper, please ask.

Name:_____

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(1) A right cylindrical tank is filled with water and is being **drained** at a rate of 25 cm^3/sec. The tank stands upright and has radius 10 cm (the radius is constant). How fast is the height of the water in the tank dropping? The volume of a right cylinder is given by $V = \pi r^2 h$.

(2) An object is thrown upward from a cliff that is 25 m tall with an initial velocity of 20 m/s. The acceleration of the object is given by -10 m/s^2.

(a) Find the equation for the position $s(t)$.

(b) When does the object reach its maximum position?

(c) When does the object hit the ground?

(3) Show that $f(x) = 2x - \sin x$ has exactly one zero in the interval $[-\pi, \pi]$. State any theorem(s) you use and why you can use them.

(4) A company wishes to manufacture a rectangular box **with no top** that holds 36 cubic feet. The bottom of the box needs to be a rectangle whose length is two times its width. Find the dimensions of the box that can be made from the minimum amount of packaging material (i.e. minimize surface area).

(5) Let $f(x)$ be defined as follows.

$$f(x) = \frac{-x^2 + 3x}{x^2 + 6x + 9} = \frac{x(3-x)}{(x+3)^2}$$

Its first and second derivatives are given by

$$f'(x) = \frac{-9(x-1)}{(x+3)^3}, \quad f''(x) = \frac{18(x-3)}{(x+3)^4}$$

(a) Find the equations of any asymptotes of f.

(b) Find the (open) intervals of increase/decrease of f.

(c) Find any local extrema of f (both coordinates).

$$f(x) = \frac{-x^2+3x}{x^2+6x+9} = \frac{x(3-x)}{(x+3)^2}, \quad f'(x) = \frac{-9(x-1)}{(x+3)^3}, \quad f''(x) = \frac{18(x-3)}{(x+3)^4}$$

(d) Find the (open) intervals of concavity and any inflection points (both coordinates) of f.

(e) Use the above information to sketch a graph of f (be sure to label the coordinates of any extrema or inflection points that you found).