Math 241 Midterm 2 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, and homework problems.

Previous Semester's Exam Problems

(1) Let a particle's motion be given by $s(t) = \sqrt{t - \cos t}$ for $t > 1$.

(a) Find the particle's **velocity** and **acceleration** as functions of t.

(b) What is the particle's **speed** at $t = 3\pi/2$?

(2) Show that $f(x) = 2x^3 + 3x^2 + 6x + 1$ has exactly one real root in $[-1, 0]$. Be sure to state and explain any theorems that you use.

(3) Let $f(x) = x^3 + 3x^2$

(a) Find the (open) intervals where f is increasing and where f is decreasing.

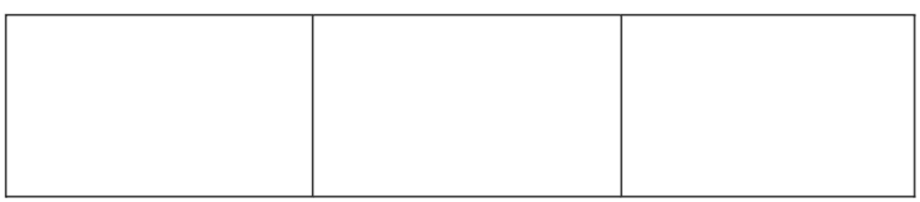
(b) Find all relative extrema (both x and y coordinates). Indicate whether it is a relative maximum or relative minimum.

(c) Find the (open) intervals where f is concave up and where f is concave down

(d) Find all inflection point(s) (both x and y coordinates)

(e) Using the information from parts (a)-(d), graph the function. Label all relative extrema and inflection point(s).

(4) An ecologist is conducting a research project on breeding pheasants in captivity. She first must construct suitable pens. She wants a rectangular area with two additional fences across its width, as shown in the sketch. Find the **dimensions** of the pen that has the maximum area she can enclose with 3600 m of fencing.



(5) A spherical snowball is placed in the sun. The sun melts the snowball so that its radius **decreases** 1/4 in. per hour. Find the rate of change of the volume with respect to time at the instant the radius is 4 in. The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

(6) A particle's acceleration is given by $a(t) = 6t + 2$. Its velocity at 1 sec is -1 m/s. Its initial position is given by $s(0) = 5$. Find the position function $s(t)$.

(7) Consider the function $f(x) = x^3 - 6x^2 + 9x$

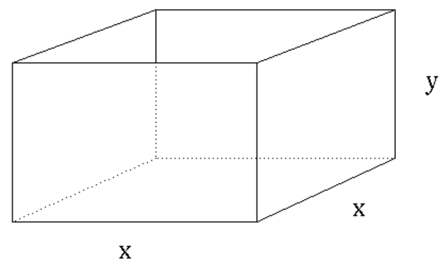
(a) Find the open intervals where f is increasing and the intervals where f is decreasing.

(b) Find both coordinates of any local extrema of the graph of f.

(c) Find the intervals where f is concave up, and the intervals where f is concave down.

(d) Find the both coordinates of any inflection point(s) of f.

(8) A box with a square base must have a volume of 8 in^3. What are the dimensions of the box that will minimize the amount of material needed to build it (i.e. minimize surface area).



(9) A person leaves a given point and travels north at 3 mph. Another person leaves the same point at the same time and travels east at 4 mph. At what rate is the distance between the two people changing at the instant when they have traveled 2 hours?

(10) Show that $x^4 - 4x = 1$ has exactly one solution on $[-1, 0]$. Please state explicitly any theorems and how you are using them.

Extra Practice Problems

(1) Use linear approximation to find the following approximations.

(a) $\frac{1}{0.9}$ given that $\frac{1}{1} = 1$

(b) $\sqrt[3]{8.5}$ given that $\sqrt[3]{8} = 2$

(c) $\frac{1.3}{1+1.3}$ given that $\frac{1}{1+1} = \frac{1}{2}$.

(2) The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12cm and the width is 5cm, find the rates of change of **a)** the area, **b)** the perimeter, and **c)** the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

(3) A rectangular plot of land will be bounded on one side by a river and on the other three sides by some sort of fence. With 800 m of fencing at your disposal, what is the largest area you can enclose, and what are its dimensions?

(4) Suppose you want to build a steel box with an open top and square base. Find the dimensions for a box of volume 500 ft^3 that will weigh as little as possible.

(5) A child flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from them at a rate of 25 ft/sec. How fast must they let out the string when the kite is 500 ft away from them?

(6) Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radios 10 cm. What is the maximum volume?

(7) A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (measured in meters) increases at a steady 10 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 3$ m?

(8) Use Newton's method to find the positive fourth root of 2 by solving the equation $x^4 - 2 = 0$. Start with $x_0 = 1$ and find x_2.

(9) Find the most general antiderivative for the following. Check your answer by differentiation.

(a) $f(x) = \frac{1}{x^2} - x^2 - \frac{1}{3}$

(b) $f(x) = 2x(1 - x^{-3})$

(10) Solve the following initial value problems.

(a) $\frac{dr}{d\theta} = -\pi \sin \pi\theta,\ r(0) = 0$

(b) $\frac{d^3y}{dx^3} = 6;\ y''(0) = -8,\ y'(0) = 0,\ y(0) = 5$

(11) Explain why $g(t) = \sqrt{t} + \sqrt{1+t} - 4$ has exactly one solution in the interval $(0, \infty)$. State any theorems used.

(12) For the following functions, **a)** find the critical points, **b)** classify them as local maxima, local minima, or neither, **c)** find where the function is increasing, **d)** find where the function is concave up, and **e)** sketch the graph.

(a) $y = x^4 - 2x^2$

(b) $y = x^5 - 5x^4$

(13) Find the absolute maximum and minimum values of the following functions of the given intervals.

(a) $f(x) = x^2 - 1, \ -1 \le x \le 2$

(b) $f(x) = \sqrt[3]{x}, \ -1 \le x \le 8$