

Sections 3.3-3.5, 7.1-7.5

Differentiation Formulas:

- $\frac{d}{dx}(c) = 0$, where c is a constant (i.e a number or a variable independent from x)
- $\frac{d}{dx}(cf(x)) = cf'(x)$
- $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
- $\frac{d}{dx}(x^n) = nx^{n-1}$

Brainstorm: Prove any of the differentiation formulas above using the limit definition of a derivative.

Exercise: Find the derivatives of the following functions:

(1) $f(x) = x^6$

(2) $y = x^{1000}$

(3) $y = r^3$

(4) $g(x) = x^\pi$

(5) $y = \frac{1}{x}$

(6) $y = \frac{1}{\sqrt[3]{x^2}}$

(7) $f(x) = \sqrt{x} - \frac{5}{x^4}$

(8) $h(x) = x^8 + 12x^5 - 4x^4 + 5$

(9) $f(x) = 2x^2 + \pi^3$

Exercise: Find the points on $y = x^4 - 6x^2 + 4$ where the line tangent to the curve is horizontal.

Exercise: The equation for the motion of a particular particle is $s(t) = 2t^3 - 5t^2 + 3t + 4$, where s is measured in meters and t in seconds. What is the acceleration after 2 seconds?

Product Rule:

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

Exercise: Find the derivatives of the following functions:

(1) $f(x) = (6x^3)(7x^4)$

(3) $g(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3)$

(2) $f(x) = (3x^2 + 1)(2x - 1)$

Quotient Rule:

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Exercise: Find the derivatives of the following functions:

(1) $y = \frac{x^2+x-2}{x^3+6}$

(3) $g(t) = \frac{\sqrt[3]{t}}{t-3}$

(2) $f(x) = \frac{3x^2+2\sqrt{x}}{x}$

(4) $f(x) = \frac{(3x^2+1)(2x-1)}{5x+4}$

Exercise: Find the equation of the line tangent to $y = \frac{\sqrt{x}}{1+x^2}$ at $(1, \frac{1}{2})$

More Differential Formulas:

- $\frac{d}{dx}(a^x) = (\ln a)a^x$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\log_a x) = \frac{d}{dx}(\log_a |x|) = \frac{1}{(\ln a)x}$
- $\frac{d}{dx}(\ln x) = \frac{d}{dx}(\ln |x|) = \frac{1}{x}$

Exercise: Find the derivatives of the following functions:

(1) $f(x) = x^4 + 4^x$

(3) $f(x) = \frac{e^x}{\ln x}$

(2) $f(x) = \log_2 x$

(4) $f(x) = x^2 e^x$

Even More Differential Formulas:

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\csc^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\csc x) = -\csc x \cot x$

Exercise: Find the derivative of $f(x) = \frac{\sec x}{1+\tan x}$

Exercise: Find the 27th derivative of $\cos x$

Exercise: Find an equation of the line tangent to $y = 2x \sin x$ at $(\frac{\pi}{2}, \pi)$

Exercise: If the position of an object is given by $s(t) = 4t \cos t$, find the equation for its acceleration.