Sections 3.9, 12.1-12.3

Related Rates (Applied Implicit Differentiation) Steps:

(1) Identify the given information and what is unknown. Draw a picture if necessary.

(2) Write down relevant equations.

(3) Differentiate with respect to the independent variable (usually t for time).

(4) Plug in your given numbers (paying attention to signs!) and solve for the unknown variable.

(5) Interpret your answer with proper units.

Brainstorm: What are some words/phrases that might indicate that a quantity is getting smaller or bigger over time (i.e. that the rate is negative or positive)?

Exercises:

(1) Air is being pumped into a spherical balloon so its volume increases at a rate of 100 cm^3/s. How fast is the radius increasing when the diameter is 50 cm?

(2) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6ft from the wall?

(3) A water tank has the shape of an inverted circular cone with base radius of 2 m and height of 4 m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.

(4) Car A is traveling west at 50 mph and Car B is traveling north at 60 mph. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when Car A is 0.3 miles and Car B is 0.4 miles from the intersection?

(5) A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the light?

Recall:

(1) Polar coordinates are coordinates written in the form of (r, θ) where r is the distance from the origin and θ is an angle whose initial side is the positive x axis and whose terminal side lies on the ray passing through the origin and point.

(2) A general polar equation is typically given in the form of $r = f(\theta)$.

(3) The equations used to convert between a polar coordinate/equation and rectangular coordinate/equation are:

- $x = r\cos\theta$
- $y = r\sin\theta$
- $x^2 + y^2 = r^2$
- $\tan\theta = \frac{y}{x}$

Brainstorm: Draw a picture to describe the above information.

Calculus with Polar Coordinates: To find $\frac{dy}{dx}$ for a polar equation $r = f(\theta)$ we use:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

Brainstorm: Where does the above formula come from?

Exercises:

(1) Find the equation for the slope of the lines tangent to the circle $r = 10$.

(2) Find the points on the interval $-\pi \leq \theta \leq \pi$ at which the cardioid $r = 1 - \cos\theta$ might have a vertical or horizontal tangent line.

Definitions:

- Parametric equations are sets of equations that express x and y in terms of a third variable (usually t for time) called the parameter.

$$x = f(t),\ y = g(t)$$

- A parametric curve consists of the points $(x, y) = (f(t), g(t))$

Exercise: Graph the parametric equations: $x = 2t$, $y = \frac{1}{2}t^2 - 4$, for $0 \le t \le 8$

Derivatives of Parametric Curves:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$$

Exercises: Find $\frac{dy}{dx}$ for the following and determine the points (if any) at which the curve has a horizontal or vertical tangent line.

(1) $x = t$, $y = 2\sqrt{t}$ for $t \ge 0$

(2) $x = 4\cos t$, $y = 16\sin t$, for $0 \le t \le 2\pi$