Sections 4.4-4.6

Steps for Curve Sketching:

(1) Find the x and y intercepts of the function

(2) Check for symmetry

   (a) If $f(-x) = f(x)$, the graph has y-axis symmetry
   (b) If $f(-x) = -f(x)$, the graph has origin symmetry

(3) Determine the domain and the location of any asymptotes

(4) Use the first derivative to find intervals of increase, intervals of decrease, and the location of any local extrema

(5) Use the second derivative to find intervals of concavity and the location of any inflection points

(6) Sketch the curve using the above information

Exercises: Graph the following functions

(1) $y = x^3 + 3x^2$ (2) $y = \frac{2x^2}{x^2-1}$ (3) $f(x) = \frac{x^2}{\sqrt{x+1}}$

(space for graphing)

(space for graphing)

Definition: An optimization problem is a word problem in which you want to maximize or minimize a certain quantity. In other words, you want to find the global maximum or global minimum of the function that describes your situation.

Theorem: Suppose c is a critical number of a continuous function f, if $f(c)$ is the only relative extremum, then it must also be the absolute extremum of f.

Brainstorm: Why does the above theorem make sense?

Steps for Solving Optimization Problems:

(1) Read the problem and draw a picture if necessary.

(2) Determine the relevant equations. Typically there are two: the objective equation and the constraint equation.

(3) Use the constraint equation to rewrite the objective equation in terms of one variable.

(4) Use a derivative test to find the absolute maximum or absolute minimum

Exercises:

(1) A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the filed that has the largest area?

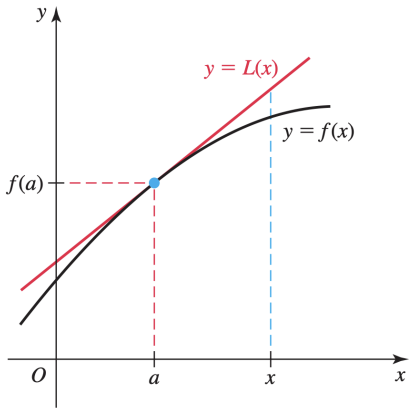
(2) A cylindrical can is to be made to hold 1 liter of oil (note: 1 L = 1000cm^3). Find the dimensions that will minimize the cost of the material needed to make the can.

(3) Find the point on the parabola $y^2 = 2x$ that is closest to $(1, 4)$

(4) A man launches his boat from point A on the bank of a straight, 3 km wide river, and wants to reach a point B that is 8 km downstream as quickly as possible. He could row his boat directly across the river and then run to B, row directly to B, or row to some point between and then run the rest of the way. If he can row 6 km/h and run 8 km/h, what should his strategy be?

(5) Find the area of the largest rectangle that can be inscribed in a semi-circle of radius 4.

Definition: Linear approximation is the process of using a line to estimate a function value.



Brainstorm: Write the equation of the line shown above.

Linearization Equation: $f(x) \approx L(x) = f(a) + f'(a)(x - a)$

Examples:

(1) Find the linearization of $f(x) = \sqrt{x+3}$ at $a = 1$. Use it to approximate $\sqrt{3.98}$ and $\sqrt{4.05}$. Are these over estimates or under estimates?

(2) Use linear approximation to approximate $\sqrt[3]{65}$

(3) Use linear approximation to approximate $\sqrt{27}$

Definition: Let f be a differentiable function. A small change in x is denoted by the differential dx. The corresponding change in y can be approximated by the differential $dy = f'(x)dx$. In other words

$$\Delta y = f(x + dx) - f(x) \approx dy = f'(x)dx$$

Brainstorm: Draw a picture to make sense of this definition.

Exercise: Compare Δy and dy if $y = x^3 + x^2 - 2x + 1$ and x changes from 2 to 2.05.