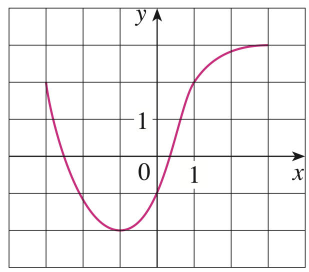
ACMAT161 Summer 2024
Professor Manguba-Glover
Classwork 1 & 2 Name: ________________________________

Complete as many of the following problems as you can with your table in the allotted time. You do not have to go in order.

Classwork 1

1. The graph of a function f is given below. Use it to answer the following questions:



   (a) What is $f(-1)$?

   (b) For what values of x is $f(x) = 2$?

   (c) State the domain and range of f.

Solution

   (a) We want to find the y-value that corresponds to $x = -1$, so that is $\boxed{-2}$

   (b) We want to find the x-values that have a y-value of 2, so that would be $\boxed{-3}$ and $\boxed{1}$

   (c) The domain is the x-values on the graph which is $\boxed{[-3, 3]}$ and the range is the y-values which is $\boxed{[-2, 3]}$

$\square$

2. If $f(x) = x^3$, evaluate $\frac{f(2+h)-f(2)}{h}$

Solution

$$\begin{aligned}
\frac{f(2+h)-f(2)}{h} &= \frac{(2+h)^3-(2)^3}{h} \\
&= \frac{(2+h)(2+h)(2+h)-8}{h} \\
&= \frac{(4+4h+h^2)(2+h)-8}{h} \\
&= \frac{\cancel{8}+12h+6h^2+h^3-\cancel{8}}{h} \\
&= \frac{12h+6h^2+h^3}{h} \\
&= \frac{\cancel{h}(12+6h+h^2)}{\cancel{h}} \\
&= \boxed{12+6h+h^2}
\end{aligned}$$

□

3. Find the domain of $f(x) = \frac{2x+1}{x^2+x-2}$

Solution We cannot divide by 0. Setting the denominator equal to zero, we get:

$$\begin{aligned}
x^2+x-2=0 &\Rightarrow (x+2)(x-1)=0 \\
&\Rightarrow x+2=0 \text{ or } x-1=0 \\
&\Rightarrow x=-2 \text{ or } x=1
\end{aligned}$$

So our domain must not include $x = -2, 1$. This gives us $\boxed{(-\infty,-2)\cup(-2,1)\cup(1,\infty)}$

□

4. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find the following:

(a) $f \circ g$ (b) $g \circ f$

Solution

(a)

$$\begin{aligned}
(f\circ g)(x) &= f(g(x)) \\
&= f(2x-3) \\
&= (2x-3)^2+2(2x-3)-1 \\
&= 4x^2-12x+9+4x-6-1 \\
&= \boxed{4x^2-8x+2}
\end{aligned}$$

(b)

$$\begin{aligned}(g\circ f)(x) &= g(x^2+2x-1)\\ &= 2(x^2+2x-1)-3\\ &= 2x^2+4x-2-3\\ &= \boxed{2x^2+4x-5}\end{aligned}$$

□

5. Find an equation for the line that passes through the point $(2,-5)$ and:

(a) Has slope -3

(b) is parallel to the x-axis

(c) is parallel to the y-axis

(d) is parallel to the line $2x-4y=3$

Solution

(a) Using point-slope form:

$$\begin{aligned}y-(-5)=-3(x-2) &\Leftrightarrow y+5=-3x+6\\ &\Leftrightarrow \boxed{y=-3x+1}\end{aligned}$$

(b) Parallel to the x-axis means that it's a horizontal line, i.e. has a slope of 0.

$$y-(-5)=0(x-2)\Leftrightarrow y+5=0\Leftrightarrow \boxed{y=-5}$$

You could also use the fact that horizontal lines have the equation $y=\#$ and since we know it goes through the point $(2,-5)$, it must have equation $y=-5$

(c) Parallel to the y-axis means that it's a vertical line, which has an undefined slope and an equation of $x=\#$. Since the line goes through the point $(2,-5)$, we know it must be $\boxed{x=2}$

(d) We have to figure out the slope for $2x-4y=3$. The easiest way to do this is to solve for y to put the equation in slope-intercept form.

$$\begin{aligned}2x-4y=3 &\Leftrightarrow -4y=-2x+3\\ &\Leftrightarrow y=\frac{-2}{-4}x+\frac{3}{-4}\\ &\Leftrightarrow y=\frac{1}{2}x-\frac{3}{4}\end{aligned}$$

Parallel lines have the same slope, so we know our line must have slope $\frac{1}{2}$ and go through the point $(2,-5)$. Using point-slope form, we have:

$$\begin{aligned}y-(-5)=\frac{1}{2}(x-2) &\Leftrightarrow y+5=\frac{1}{2}x-1\\ &\Leftrightarrow \boxed{y=\frac{1}{2}x-6}\end{aligned}$$

□

6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

(a) Evaluate $f(-2)$ and $f(1)$

(b) Sketch the graph of f

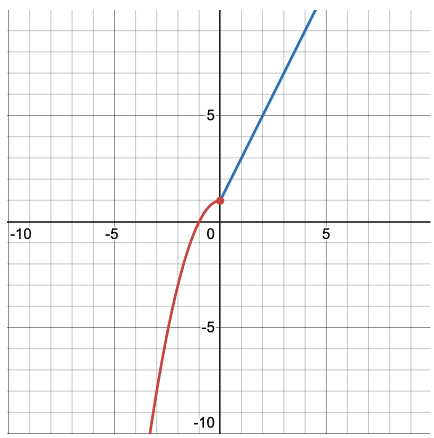
Solution

(a) We want to find the correct piece for each input. $-2 \leq 0$, so we are going to use the top piece for $f(-2)$. $1 > 0$, so we are going to use the bottom piece for $f(1)$

$$\begin{aligned} f(-2) &= 1 - (-2)^2 \\ &= 1 - 4 \\ &= \boxed{-3} \end{aligned}$$

$$\begin{aligned} f(1) &= 2(1) + 1 \\ &= \boxed{3} \end{aligned}$$

(b) We know that our graph looks like the graph of $1 - x^2$ for $x \leq 0$ and $2x + 1$ for $x >$. Putting these two things together, we get:



□

Classwork 2

1. Simplify $(3a^3b^3)(4ab^2)^2$

   Solution

$$\begin{aligned}(3a^3b^3)(4ab^2)^2 &= (3a^3b^3)(16a^2b^4)\\ &= (3\cdot 16)(a^3a^2)(b^3b^4)\\ &= \boxed{48a^5b^7}\end{aligned}$$

□

2. Simplify $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

   Solution

$$\begin{aligned}\left(\frac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2} &= \left(3x^{3/2-2}y^{3-(-1/2)}\right)^{-2}\\ &= (3x^{3/2-4/2}y^{6/2+1/2})^{-2}\\ &= (3x^{-1/2}y^{7/2})^{-2}\\ &= 3^{-2}x^{-1/2\cdot(-2)}y^{7/2\cdot-2}\\ &= \frac{1}{3^2}x^1y^{-7}\\ &= \frac{1}{9}\cdot x\cdot\frac{1}{y^7}\\ &= \boxed{\frac{x}{9y^7}}\end{aligned}$$

□

3. Solve $e^{5-3x} = 10$

   Solution

$$\begin{aligned}e^{5-3x} = 10 &\Rightarrow \ln(e^{5-3x}) = \ln 10\\ &\Leftrightarrow 5-3x = \ln 10\\ &\Leftrightarrow -3x = \ln 10 - 5\\ &\Leftrightarrow x = -\frac{1}{3}(\ln 10 - 5)\\ &\Leftrightarrow x = \boxed{\frac{1}{3}(5-\ln 10)}\end{aligned}$$

□

4. Solve $-6\log_3(x-3) = -24$

Solution

$$\begin{aligned} -6\log_3(x-3) = -24 &\Leftrightarrow \log_3(x-3) = 4 \\ &\Rightarrow 3^{\log_3(x-3)} = 3^4 \\ &\Rightarrow x - 3 = 81 \\ &\Leftrightarrow x = \boxed{84} \end{aligned}$$

□

5. Expand: $\log_5\left(\frac{\sqrt{x}}{25y^5}\right)$

Solution

$$\begin{aligned} \log_5\left(\frac{\sqrt{x}}{25y^5}\right) &\Leftrightarrow \log_5\sqrt{x} - (\log_5 25 + \log_5 y^5) \\ &\Leftrightarrow \log_5\sqrt{x} - \log_5 25 - \log_5 y^5 \\ &\Leftrightarrow \log_5 x^{1/2} - \log_5(5^2) - \log_5 y^5 \\ &= \boxed{\frac{1}{2}\log_5 x - 2 - 5\log_5 y} \end{aligned}$$

□

6. Combine into one logarithm: $5\ln c - \ln k + 4\ln y$

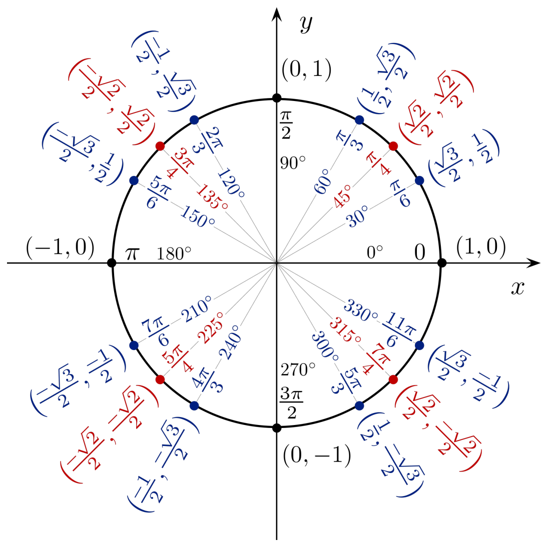
Solution

$$\begin{aligned} 5\ln c - \ln k + 4\ln y &= 5\ln c + 4\ln y - \ln k \\ &= \ln c^5 + \ln y^4 - \ln k \\ &= \boxed{\ln\left(\frac{c^5y^4}{k}\right)} \end{aligned}$$

□

7. Find $\tan\left(\frac{\pi}{3}\right)$, $\sin\left(\frac{7\pi}{6}\right)$, and $\sec\left(\frac{5\pi}{3}\right)$

Solution This problem is just testing your knowledge of the unit circle. All of these values can be found on the standard version.



$$\tan\left(\frac{\pi}{3}\right) = \frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$$

$$= \boxed{\sqrt{3}}$$

$$\sin\left(\frac{7\pi}{7}\right) = \boxed{-\frac{1}{2}}$$

$$\sec\left(\frac{5\pi}{3}\right) = \frac{1}{\cos\left(\frac{5\pi}{3}\right)}$$

$$= \frac{1}{\frac{1}{2}}$$

$$= \boxed{2}$$

□

8. Solve $2\sin^2\theta + 5\sin\theta = 3$

Solution

$$\begin{aligned} 2\sin^2\theta + 5\sin\theta = 3 &\Leftrightarrow 2\sin^2\theta + 5\sin\theta - 3 = 0 \\ &\Leftrightarrow (2\sin\theta - 1)(\sin\theta + 3) = 0 \\ &\Leftrightarrow 2\sin\theta - 1 = 0 \text{ or } \sin\theta + 3 = 0 \\ &\Leftrightarrow \sin\theta = \frac{1}{2} \text{ or } \sin\theta = -3 \\ &\Leftrightarrow \theta = \boxed{\frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n} \end{aligned}$$

□

9. Evaluate $\sin^{-1}\left(\frac{1}{2}\right)$

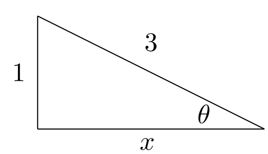
Solution This is once again a test of how well you know the unit circle. Keep in mind that inverse sine is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. We want to find the angle in that section where $\sin\theta = \frac{1}{2}$. That gives us $\boxed{\frac{\pi}{6}}$

□

10. Evaluate $\tan\left(\arcsin\frac{1}{3}\right)$

Solution $\frac{1}{3}$ is not a value on the unit circle, but it is between −1 and 1. To solve this problem, we can draw a triangle that contains and angle θ such that $\sin\theta = \frac{1}{3}$.

$$\sin^{-1}\left(\frac{1}{3}\right) = \theta \Rightarrow \sin\theta = \frac{1}{3} = \frac{\text{opposite}}{\text{hyoptenuse}}$$



Solving for the missing side using the pythagorean theorem:

$$\begin{aligned} 1^2 + x^2 = 3^2 &\Rightarrow 1 + x^2 = 9 \\ &\Rightarrow x^2 = 8 \\ &\Rightarrow x = \sqrt{8} \\ &\Rightarrow x = 2\sqrt{2} \end{aligned}$$

Finally, we have

$$\tan\left(\sin^{-1}\left(\frac{1}{3}\right)\right) = \tan\theta = \boxed{\frac{1}{2\sqrt{2}}}$$

□