ACMAT161 Summer 2024
Professor Manguba-Glover
Classwork 13 & 14 Name: _____

Complete as many of the following problems as you can with your table in the allotted time. You do not have to go in order.

Classwork 13

1. Suppose that x and y are two functions of t such that the following equation holds for all t:

$$\sin(x+y) + (1+x)^2 + (1+y)^2 = 5$$

   and suppose that when $x = 0$ and $y = 0$ we have $dx/dt = -10$. What is the value of dy/dt?

2. A man 6 feet tall is walking away from a lamp post at the rate of 50 feet per minute. When the man is 8 feet away from the post his shadow is 10 feet long. Find the rate at which the length of the shadow is increasing when he is 25 feet away from the post.

3. One car leaves a given point and travels north at 30 mph. Another car leaves the same point at the same time and travels west at 40 mph. At what rate is the distance between the two cars changing at the instant when the cars have traveled 2 hours?

4. Consider a bacterial culture growing in a petri dish and suppose that the bacteria reproduce in such a way that they are always forming a disk of growing radius. Suppose that we know the area of the disk is increasing at the rate of 3 cm^2/day. Can we find the rate of change of the radius at the time when that radius is 4 cm?

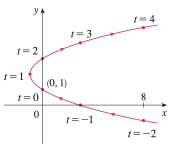
Key:

1. 10
2. 62.5 ft/min
3. 50 mph
4. $\frac{3}{8\pi}$ cm/day

Classwork 14

1. Find the slope of the tangent line to $r = 1 + \sin\theta$ when $\theta = \frac{\pi}{3}$
2. Find the derivative of $r = 3 + 2\cos\theta$
3. Sketch the curve defined by $x = t^2 - 2t$, $y = t + 1$
4. Find an equation of the tangent line to the curve $x = \sec t$, $y = \tan t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$ when $t = \frac{\pi}{4}$
5. A curve is defined by $x = t^2$, $y = t^3 - 3t$. Find $\frac{dy}{dx}$ and determine where the tangent is horizontal or vertical.
6. Find the equation of the slope of the line tangent to $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$.

Key:

1. -1
2. $-\frac{2\cos(2\theta)+3\cos\theta}{2\sin(2\theta)+3\sin\theta}$
3. 

4. $y - 1 = \sqrt{2}(x - \sqrt{2})$
5. $\frac{3t^2-3}{2t}$, $(1,-2)$, $(1,2)$, $(0,0)$
6. $\frac{\sin\theta}{1-\cos\theta}$