ACMAT161 Summer 2024
Professor Manguba-Glover
Classwork 15 & 16 Name: ________________________________

Complete as many of the following problems as you can with your table in the allotted time. You do not have to go in order.

Classwork 15

1. Find the absolute extrema of $f(x) = x^4 - 2x^3$ on $[-2, 2]$
2. Find the absolute extrema of $g(x) = x^{2/3}(2 - x)$ on $[-1, 2]$
3. Find the absolute extrema of $y = x^2$ on $[-2, 1]$
4. Find the absolute extrema of $f(x) = 10x(2 - \ln x)$ on $[1, e^2]$
5. Show that $x^3 + 3x + 1 = 0$ has exactly one solution.
6. Show that $2x + \cos x = 0$ has exactly one solution.

Key:

1. $f(-2) = 32$ and $f\left(\frac{3}{2}\right) = -\frac{27}{16}$
2. $g(-1) = 3$ and $g(0) = g(2) = 0$
3. $(-2, 4)$ and $(0, 0)$
4. $f(e) = 10e$ and $f(e^2) = 0$
5. Use IVT and MVT/Rolle's
6. Use IVT and MVT/Rolle's

Classwork 16

1. Sketch a graph of a function f that is continuous and satisfies the following
   - $f' > 0$ on $(-\infty, 0)$, $(4, 6)$, and $(6, \infty)$
   - $f' < 0$ on $(0, 4)$
   - $f'(0)$ is undefined
   - $f'(4) = f'(6) = 0$
2. Find the interval(s) on which the function is increasing and on which it is decreasing: $f(x) = 2x^3 + 3x^2 + 1$
3. Find the interval(s) on which the function is increasing and on which it is decreasing: $f(x) = xe^{-x}$
4. Find the interval(s) on which f is increasing, on which it is decreasing, and find any local extrema: $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$
5. Identify where the function is concave up and where it is concave down. Find the location of any inflection points: $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$
6. Identify where the function is concave up and where it is concave down. Find the location of any inflection points: $f(x) = \sin^{-1} x$ on $(-1, 1)$

Key:

1. Many possible answers
2. Increasing on $(-\infty, -1)$ and $(0, \infty)$,
   decreasing on $(-1, 0)$
3. Increasing on $(-\infty, 1)$
   decreasing on $(1, \infty)$
4. Increasing on $(-1, 1)$ and $(1, \infty)$
   decreasing on $(-\infty, -1)$, local max of $f(-1) = -10$
5. Concave up on $\left(-\infty, -\frac{1}{3}\right)$, $(1, \infty)$
   concave down on $\left(-\frac{1}{3}, 1\right)$
   inflection points at $x = 1$ and $x = -\frac{1}{3}$
6. Concave up on $(0, 1)$, concave down on $(-1, 0)$
   inflection point at $x = 0$