ACMAT161 Summer 2024
Professor Manguba-Glover
Classwork 15 & 16 Name: ______________________________

Complete as many of the following problems as you can with your table in the allotted time. You do not have to go in order.

Classwork 15

(1) Find the absolute extrema of $f(x) = x^4 - 2x^3$ on $[-2, 2]$

Solution

$$f'(x) = 4x^3 - 6x^2 = 2x^2(2x - 3)$$

Setting this equal to 0 and solving, we get $x = 0, \frac{3}{2}$

$$f(-2) = 32 \leftarrow \text{abs max}$$

$$f(0) = 0$$

$$f\left(\frac{3}{2}\right) = -\frac{27}{16} \leftarrow \text{abs min}$$

$$f(2) = 0$$

□

(2) Find the absolute extrema of $g(x) = x^{2/3}(2 - x)$ on $[-1, 2]$

Solution

$$g'(x) = \frac{4}{3}x^{-1/3} - \frac{5}{3}x^{2/3} = \frac{4 - 5x}{3x^{1/3}}$$

This is zero at $x = \frac{4}{5}$ and undefined at $x = 0$

$$g(-1) = 3 \leftarrow \text{abs max}$$

$$g(0) = 0 \leftarrow \text{abs min}$$

$$g\left(\frac{4}{5}\right) \approx 1.03$$

$$g(2) = 0 \leftarrow \text{abs min}$$

□

(3) Find the absolute extrema of $y = x^2$ on $[-2, 1]$

Solution

$$\frac{dy}{dx} = 2x$$

So the only critical value is $x = 0$

$$f(-2) = 4 \leftarrow \text{abs max}$$

$$f(0) = 0 \leftarrow \text{abs min}$$

$$f(1) = 1$$

□

(4) Find the absolute extrema of $f(x) = 10x(2 - \ln x)$ on $[1, e^2]$

Solution

$$f'(x) = 10(2 - \ln x) + 10x\left(-\frac{1}{x}\right) = -(10lnx - 1)$$

So the only critical number is $x = e$

$$f(1) = 20$$

$$f(e) = 10e \leftarrow \text{abs max}$$

$$f(e^2) = 0 \leftarrow \text{abs min}$$

□

(5) Show that $x^3 + 3x + 1 = 0$ has exactly one solution.

Solution Let $f(x) = x^3 + 3x + 1$,

$$f(0) = 1 \text{ and } f(-1) = -3$$

So by IVT, the equation has at least one solution. If there were a second solution, then by MVT, $f'(x) = 0$ at some point.

$$f'(x) = 3x^2 + 3 = 3(x^2 + 1)$$

which has no real roots, therefore the equation has exactly one solution.

□

(6) Show that $2x + \cos x = 0$ has exactly one solution.

Solution Let $f(x) = 2x + \cos x$

$$f\left(\frac{\pi}{2}\right) = \pi \text{ and } f\left(-\frac{\pi}{2}\right) = -\pi$$

So by IVT, the equation has at least one solution. If there were a second solution, then by MVT, $f'(x) = 0$ at some point.

$$f'(x) = 2 - \sin x$$

$\sin x$ can never equal 2, so there must be exactly one solution.

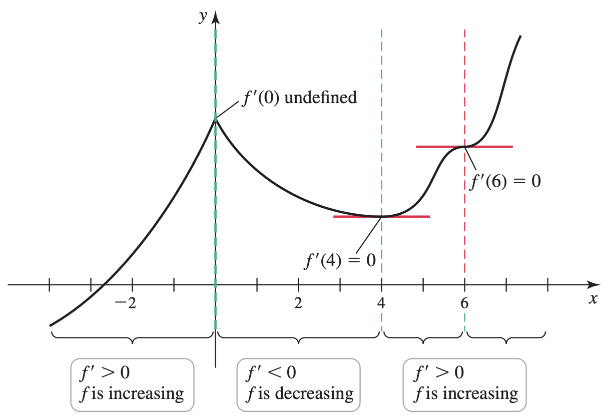
□

Classwork 16

(1) Sketch a graph of a function f that is continuous and satisfies the following

- $f' > 0$ on $(-\infty, 0)$, $(4, 6)$, and $(6, \infty)$
- $f' < 0$ on $(0, 4)$
- $f'(0)$ is undefined
- $f'(4) = f'(6) = 0$

Solution



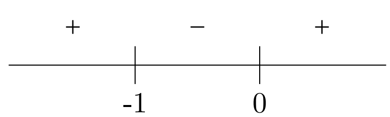
□

(2) Find the interval(s) on which the function is increasing and on which it is decreasing: $f(x) = 2x^3 + 3x^2 + 1$

Solution

$$f'(x) = 6x + 6 = 6x(x + 1)$$

Our critical points at $x = 0, -1$. Plugging in and testing we get:



Therefore f is increasing on $(-\infty, -1)$ and $(0, \infty)$, and it is decreasing on $(-1, 0)$

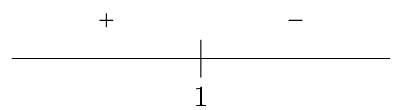
□

(3) Find the interval(s) on which the function is increasing and on which it is decreasing: $f(x) = xe^{-x}$

Solution

$$f'(x) = e^{-x} + xe^{-x} = \frac{1-x}{e^x}$$

So our critical number is just $x = 1$



Therefore, f is increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$

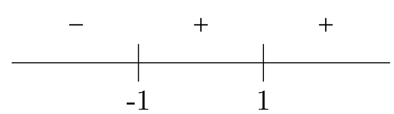
$\square$

(4) Find the interval(s) on which f is increasing, on which it is decreasing, and find any local extrema: $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$

Solution

$$f'(x) = 12x^3 - 12x^2 - 12x + 12 = 12(x+1)(x-1)^2$$

We have critical numbers of -1 and 1. Plotting on a number line gives



Thus, f is increasing on $(-1, 1)$ and $(1, \infty)$ and decreasing on $(-\infty, -1)$. We also know that there is a local maximum at $x = -1$ with

$$f(-1) = 3(1) - 4(-1) - 6(1) + 12(-1) + 1 = -10$$

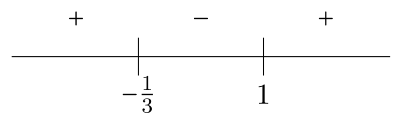
$\square$

(5) Identify where the function is concave up and where it is concave down. Find the location of any inflection points: $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$

Solution

$$f'(x) = 12x^3 - 12x^2 - 12x + 12 \Rightarrow f''(x) = 36x^2 - 24x - 12 = 12(x-1)(3x+1)$$

So our numbers to plot are $x = 1$ and $x = -\frac{1}{3}$.



So, f is concave up on $\left(-\infty, -\frac{1}{3}\right)$, $(1, \infty)$ and concave down on $\left(-\frac{1}{3}, 1\right)$. this also means that there are two inflection points at $x = -\frac{1}{3}$ and $x = 1$.

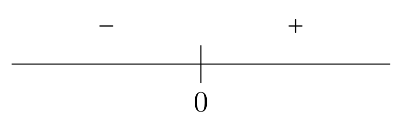
□

(6) Identify where the function is concave up and where it is concave down. Find the location of any inflection points: $f(x) = \sin^{-1} x$ on $(-1, 1)$

Solution

$$f'(x) = \frac{1}{\sqrt{1-x^2}} \text{ and } f''(x) = -\frac{1}{2}(1-x^2)^{-3/2} \cdot -2x = \frac{x}{(1-x^2)^{3/2}}$$

The domain of $\sin^{-1}$ is $[-1, 1]$ so the only number to plot is $x = 0$, we get



This gives that f is concave up on $(0, 1)$ and concave down on $(-1, 0)$ with an inflection point at $x = 0$.

□