ACMAT161 Summer 2024
Professor Manguba-Glover
Classwork 19 & 20 Name: ________________________________

Complete as many of the following problems as you can with your table in the allotted time. You do not have to go in order.

Classwork 19

(1) Evaluate $\lim_{x \to 1} \frac{\ln x}{x-1}$

Solution

$$\lim_{x \to 1} \frac{\ln x}{x-1} \to \frac{0}{0} \text{ Indeterminate Form}$$

$$\begin{aligned} \lim_{x \to 1} \frac{\ln x}{x-1} &\overset{L'H}{=} \lim_{x \to 1} \frac{\frac{1}{x}}{1} \\ &= \lim_{x \to 1} \frac{1}{x} \\ &= \frac{1}{1} \\ &= \boxed{1} \end{aligned}$$

$\square$

(2) Evaluate $\lim_{x \to \infty} \frac{e^x}{x^2}$

Solution

$$\lim_{x \to \infty} \frac{e^x}{x^2} \to \frac{\infty}{\infty}$$

$$\begin{aligned} \lim_{x \to \infty} \frac{e^x}{x^2} &\overset{L'H}{=} \lim_{x \to \infty} \frac{e^x}{2x} \left(\to \frac{\infty}{\infty} \right) \\ &\overset{L'H}{=} \lim_{x \to \infty} \frac{e^x}{2} \\ &\to \frac{\infty}{2} \\ &\to \boxed{\infty} \end{aligned}$$

$\square$

(3) Evaluate $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

$$\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} \to \frac{\infty}{\infty} \text{ Indefinite Form}$$

$$\begin{aligned}
\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} &\overset{L'H}{=} \lim_{x\to\infty} \frac{1/x}{1/2 \cdot x^{-1/2}} \\
&= \lim_{x\to\infty} \frac{1/x}{\frac{1}{2\sqrt{x}}} \\
&= \lim_{x\to\infty} \frac{1}{x} \cdot \frac{2\sqrt{x}}{1} \\
&= \lim_{x\to\infty} \frac{2}{\sqrt{x}} \\
&\to \frac{2}{\infty} \\
&\to \boxed{0}
\end{aligned}$$

(4) Evaluate $\lim_{x\to 0} \frac{\tan x - x}{x^3}$

Solution

$$\lim_{x\to 0} \frac{\tan x - x}{x^3} \to \frac{0}{0} \text{ Indefinite Form}$$

$$\begin{aligned}
\lim_{x\to 0} \frac{\tan x - x}{x^3} &\overset{L'H}{=} \lim_{x\to 0} \frac{\sec^2 x - 1}{3x^2} \left(\to \frac{0}{0}\right) \\
&\overset{L'H}{=} \lim_{x\to 0} \frac{2\sec x \cdot \sec x \tan x}{6x} \\
&= \lim_{x\to 0} \frac{2\sec^2 x \tan x}{6x} \left(\to \frac{0}{0}\right) \\
&\overset{L'H}{=} \lim_{x\to 0} \frac{4\sec x \cdot \sec x \tan x \cdot \tan x + 2\sec^2 x \sec^2 x}{6} \\
&= \lim_{x\to 0} \frac{4\sec^2 x \tan^2 x + 2\sec^3 x}{6} \\
&= \frac{4(1)^2(0)^2 + 2(1)^3}{6} \\
&= \frac{2}{6} \\
&= \boxed{\frac{1}{3}}
\end{aligned}$$

□

(5) Evaluate $\lim\limits_{x\to 0^+} x\ln x$

Solution

$$\lim_{x\to 0^+} x\ln x \to 0\cdot -\infty \text{ Indefinite Form}$$

$$\begin{aligned}
\lim_{x\to 0^+} x\ln x &= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \left(\to \frac{-\infty}{\infty}\right) \\
&\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} \\
&= \lim_{x\to 0^+} \frac{1}{x}\cdot\frac{-x^2}{1} \\
&= \lim_{x\to 0^+} -\frac{x^2}{x} \\
&= \lim_{x\to 0^+} -x \\
&= \boxed{0}
\end{aligned}$$

□

(6) Evaluate $\lim\limits_{x\to \frac{\pi}{2}^-} (\sec x - \tan x)$

Solution

$$\lim_{x\to \frac{\pi}{2}^-} (\sec x - \tan x) \to \infty - \infty \text{ Indeterminate Form}$$

$$\begin{aligned}
\lim_{x\to \frac{\pi}{2}^-} (\sec x - \tan x) &= \lim_{x\to \frac{\pi}{2}^-} \frac{1}{\cos x} - \frac{\sin x}{\cos x} \\
&= \lim_{x\to \frac{\pi}{2}^-} \frac{1-\sin x}{\cos x} \left(\to \frac{0}{0}\right) \\
&\overset{L'H}{=} \lim_{x\to \frac{\pi}{2}^-} \frac{-\cos x}{-\sin x} \\
&= \frac{-0}{-1} \\
&= \boxed{0}
\end{aligned}$$

□

(7) Evaluate $\lim_{x\to\infty} x^{1/x}$

Solution

$$\lim_{x\to\infty} x^{1/x} \to \infty^0 \text{ Indeterminate Form}$$

Taking the natural log, we have

$$\begin{aligned}
\lim_{x\to\infty} \ln\left(x^{1/x}\right) &= \lim_{x\to\infty} \frac{1}{x}\ln x \\
&= \lim_{x\to\infty} \frac{\ln x}{x} \left(\to \frac{\infty}{\infty}\right) \\
&\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} \\
&= \lim_{x\to\infty} \frac{1}{x} \\
&= 0
\end{aligned}$$

So our answer is:

$$e^0 = \boxed{1}$$

□

Classwork 20

(1) Use Newton's Method to determine x_2 when $f(x) = x^3 - 7x^2 + 8x - 3$ and $x_0 = 5$

Solution

$$f'(x) = 3x^2 - 14x + 8$$

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 5 - \frac{f(5)}{f'(5)} \\ &= 5 - \frac{-13}{13} \\ &= 6 \end{aligned}$$

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= 6 - \frac{f(6)}{f'(6)} \\ &= 6 - \frac{9}{32} \\ &= \boxed{5.71875} \end{aligned}$$

□

(2) Find the solution to $x^4 - 5x^3 + 9x + 3 = 0$ that is in the interval $[4, 6]$

Solution Since we're between 4 and 6, we can choose an initial guess of $x_0 = 5$

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 5 - \frac{48}{134} \\ &= 4.641791045 \end{aligned}$$

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= 4.641791045 - \frac{8.950542057}{85.85891882} \\ &= 4.537543959 \end{aligned}$$

$$\begin{aligned} x_3 &= x_2 - \frac{f(x_2)}{f'(x_2)} \\ &= 4.537543959 - \frac{0.6329967413}{73.85993168} \\ &= 4.528973727 \end{aligned}$$

$$\begin{aligned} x_4 &= x_3 - \frac{f(x_3)}{f'(x_3)} \\ &= 4.528973727 - \frac{0.004066133005}{72.91199944} \\ &= 4.52891796 \end{aligned}$$

$$\begin{aligned} x_5 &= x_4 - \frac{f(x_4)}{f'(x_4)} \\ &= 4.52891796 - \frac{0.0000001714694911}{72.90585006} \\ &= 4.52891796 \end{aligned}$$

Since the approximation has stopped changing, we can say that $\boxed{x \approx 4.52891796}$

□

(3) Find the most general antiderivative of $f(x) = 3x^2$

Solution

$$3 \cdot \frac{x^{2+1}}{2+1} + C = \frac{3x^3}{3} + C = \boxed{x^3 + C}$$

□

(4) Find the most general antiderivative of $f(x) = \sin t$

Solution

$$\boxed{\cos t + C}$$

□

(5) Evaluate $\int (3x^5 + 2 - 5\sqrt{x} - \frac{1}{x})\ dx$

Solution

$$\begin{aligned}\int (3x^5 + 2 - 5\sqrt{x} - \frac{1}{x})\ dx &= \int (3x^5 + 2x^0 - 5x^{1/2} - x^{-1})\ dx \\ &= 3 \cdot \frac{x^6}{6} + 2\frac{x^1}{1} - 5 \cdot \frac{x^{3/2}}{3/2} - \ln|x| + C \\ &= \boxed{\frac{x^6}{2} + 2x - \frac{10x^{3/2}}{3} - \ln|x| + C}\end{aligned}$$

□

(6) Evaluate $\int (2x + 3\cos x + \frac{e^x}{3})\ dx$

Solution

$$\begin{aligned}\int (2x + 3\cos x + \frac{e^x}{3})\ dx &= \int \left(2x + 3\cos x + \frac{1}{3}e^x\right)\ dx \\ &= 2 \cdot \frac{x^2}{2} + 3\sin x + \frac{1}{3}e^x + C \\ &= \boxed{x^2 + 3\sin x + \frac{1}{3}e^x + C}\end{aligned}$$

□

(7) Evaluate $\int \frac{\sin x}{\cos^2 x}\ dx$

$$\begin{aligned}\int \frac{\sin x}{\cos^2 x}\ dx &= \int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\ dx\\ &= \int \sec x \tan x\ dx\\ &= \boxed{\sec x + C}\end{aligned}$$

(8) Solve the initial value problem: $f'(x) = x^2 - 2x,\ f(1) = \frac{1}{3}$

Solution

$$\begin{aligned}f(x) &= \int (x^2 - 2x)\ dx\\ &= \frac{x^3}{3} - 2\cdot\frac{x^2}{2} + C\\ &= \frac{x^3}{3} - x^2 + C\end{aligned}$$

Plugging in the fact that $f(1) = \frac{1}{3}$, and solving for C, we have:

$$\begin{aligned}f(1) = \frac{1}{3} &\Leftrightarrow \frac{(1)^3}{3} - (1)^2 + C = \frac{1}{3}\\ &\Leftrightarrow \frac{1}{3} - 1 + C = \frac{1}{3}\\ &\Leftrightarrow -1 + C = 0\\ &\Leftrightarrow C = 1\end{aligned}$$

Putting this together, we have

$$f(x) = \boxed{\frac{x^3}{3} - x^2 + 1}$$

□

(9) A particle moves in a straight line with acceleration $a(t) = 6t + 4$. Its initial velocity is -6 cm/s and its initial position is at 9 cm. Find the equation for position at time t.

$$\begin{aligned} v(t) &= \int a(t)\ dt \\ &= \int (6t+4)\ dt \\ &= 6 \cdot \frac{t^2}{2} + 4t + C \\ &= 3t^2 + 4t + C \end{aligned}$$

$$\begin{aligned} s(t) &= \int v(t)\ dt \\ &= \int (3t^2 + 4t + C)\ dt \\ &= 3 \cdot \frac{t^3}{3} + 4 \cdot \frac{t^2}{2} + Ct + D \\ &= t^3 + 2t^2 + Ct + D \end{aligned}$$

Using the fact that $v(0) = -6$, we have

$$v(0) = 3(0)^2 + 4(0) + C = -6 \Leftrightarrow C = -6$$

Using the fact that $s(0) = 9$, we have

$$s(0) = (0)^3 + 2(0)^2 - 6(0) + D = 9 \Leftrightarrow D = 9$$

Finally, we have

$$s(t) = \boxed{t^3 + 2t^2 - 6t + 9}$$