ACMAT161 Summer 2024
Professor Manguba-Glover
Classwork 21 & 22 Name: ______________________________

Complete as many of the following problems as you can with your table in the allotted time. You do not have to go in order.

Classwork 21

1. Approximate the area bounded by the graph of $f(x) = 3\sqrt{x}$ and the x-axis between $x = 4$ and $x = 16$ with 6 rectangles.

   (a) Using left endpoints

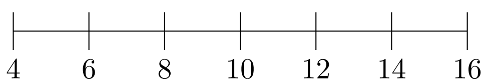
   (b) Using right endpoints

   (c) Are your answers is (a) and (b) overestimates or underestimates?

   (d) Using midpoints

   Solution

$$\Delta x = \frac{b-a}{n} = \frac{16-4}{6} = 2$$



(a)

$$\begin{aligned} L_6 &= f(4)\cdot 2 + f(6)\cdot 2 + f(8)\cdot 2 + f(10)\cdot 2 + f(12)\cdot 2 + f(14)\cdot 2 \\ &= 3\sqrt{4}\cdot 2 + 3\sqrt{6}\cdot 2 + 3\sqrt{8}\cdot 2 + 3\sqrt{10}\cdot 2 + 3\sqrt{12}\cdot 2 + 3\sqrt{14}\cdot 2 \\ &\approx \boxed{105.876} \end{aligned}$$

(b)

$$\begin{aligned} R_6 &= f(6)\cdot 2 + f(8)\cdot 2 + f(10)\cdot 2 + f(12)\cdot 2 + f(14)\cdot 2 + f(16)\cdot 2 \\ &= 3\sqrt{6}\cdot 2 + 3\sqrt{8}\cdot 2 + 3\sqrt{10}\cdot 2 + 3\sqrt{12}\cdot 2 + 3\sqrt{14}\cdot 2 + 3\sqrt{16}\cdot 2 \\ &\approx \boxed{117.876} \end{aligned}$$

(c) The graph of $3\sqrt{x}$ is concave down, so the left sums are underestimates and the right sums are overestimates.

(d) Finding the midpoints of the above sections, we have:

$$\begin{aligned} M_6 &= f(5)\cdot 2 + f(7)\cdot 2 + f(9)\cdot 2 + f(11)\cdot 2 + f(13)\cdot 2 + f(15)\cdot 2 \\ &= 3\sqrt{5}\cdot 2 + 3\sqrt{7}\cdot 2 + 3\sqrt{9}\cdot 2 + 3\sqrt{11}\cdot 2 + 3\sqrt{13}\cdot 2 + 3\sqrt{15}\cdot 2 \\ &\approx \boxed{112.062} \end{aligned}$$

$\square$

2. Estimate the area under the graph of f on the interval $[0, 2]$ using left and right Riemann sums with $n = 4$, where f is continuous and has values given by the following table:

x	$f(x)$
0	1
0.5	3
1	4.5
1.5	5.5
2	6

Solution

$$\Delta x = \frac{2}{4} = 0.5$$

$$\begin{aligned} L_4 &= f(0)\Delta x + f(0.5)\Delta x + f(1)\Delta x + f(1.5)\Delta x \\ &= 1 \cdot 0.5 + 3 \cdot 0.5 + 4.5 \cdot 0.5 + 5.5 \cdot 0.5 + 5.5 \cdot 0.5 \\ &= \boxed{7} \end{aligned}$$

$$\begin{aligned} R_4 &= f(0.5)\Delta x + f(1)\Delta x + f(1.5)\Delta x + f(2)\Delta x \\ &= 3 \cdot 0.5 + 4.5 \cdot 0.5 + 5.5 \cdot 0.5 + 5.5 \cdot 0.5 + 6 \cdot 0.5 \\ &= \boxed{9.5} \end{aligned}$$

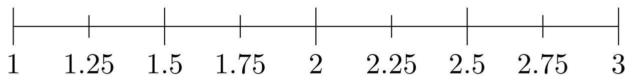
□

3. Evaluate the midpoint Riemann sum for $f(x) = 1 - x^2$ on $[1, 3]$ with $n = 4$ (Note: this function is below the x-axis, so the area will be negative)

Solution

$$\Delta x = \frac{3-1}{4} = \frac{1}{2}$$

Splitting our intervals and finding the midpoints, we have:



$$\begin{aligned} M_4 &= \frac{1}{2}(f(1.25) + f(1.75) + f(2.25) + f(2.75)) \\ &= \frac{1}{2}((1-(1.25)^2) + (1-(1.75)^2) + (1-(2.25)^2) + (1-(2.75)^2)) \\ &= \boxed{-6.625} \end{aligned}$$

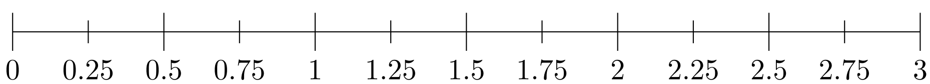
□

4. Evaluate the midpoint Riemann sum for $f(x) = 1 - x^2$ on $[0, 3]$ with $n = 6$ (Note: this function is below the x-axis, so the area will be negative)

Solution

$$\Delta x = \frac{3-0}{6} = \frac{1}{2}$$

Splitting our intervals and finding the midpoints, we have:



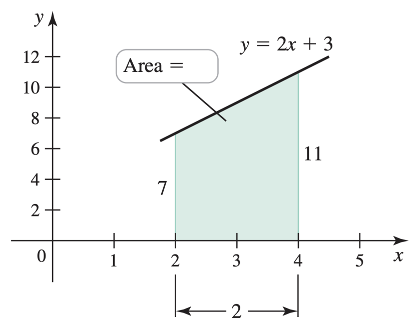
$$\begin{aligned} M_6 &= \frac{1}{2}(f(0.25) + f(0.75) + f(1.25) + f(1.75) + f(2.25) + f(2.75)) \\ &= \frac{1}{2}(1-(0.25)^2) + (1-(0.75)^2) + (1+(1.25)^2) + (1+(1.75)^2) + (1+(2.25)^2) + (1+(2.75)^2) \\ &\approx \boxed{-3.875} \end{aligned}$$

□

Classwork 22

1. Use geometry to evaluate $\int_2^4 (2x+3)\ dx$

Solution The graph of $y = 2x + 3$ from $x = 2$ to $x = 4$ looks like



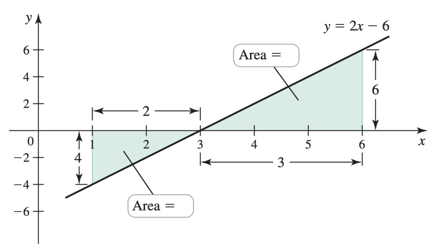
This is a trapezoid, so we can use the equation for the area of a trapezoid

$$\int_2^4 (2x+3)\ dx = \frac{1}{2} \cdot 2(11+7)$$
$$= \boxed{18}$$

$\square$

2. Use geometry to evaluate $\int_1^6 (2x-6)\ dx$ (Note: keep in mind negative and positive areas)

Solution The graph of $y = 2x - 6$ from $x = 1$ to $x = 6$ looks like



These are two triangles, one with negative area and one with positive area:

$$\int_1^6 (2x-6)\ dx = -\frac{1}{2} \cdot 2 \cdot 4 + \frac{1}{2} \cdot 3 \cdot 6$$
$$= -4 + 9$$
$$= \boxed{5}$$

$\square$

3. Assume $\int_0^5 f(x)\ dx = 3$ and $\int_0^7 f(x)\ dx = -10$. Evaluate the following:

   (a) $\int_0^7 2f(x)\ dx$

   (b) $\int_5^7 f(x)\ dx$

   (c) $\int_7^0 f(x)\ dx$

   Solution

   (a)

$$\begin{aligned}\int_0^7 2f(x)\ dx &= 2\int_0^7 f(x)\ dx \\ &= 2(-10) \\ &= \boxed{-20}\end{aligned}$$

   (b)

$$\begin{aligned}\int_0^7 f(x)\ dx = \int_0^5 f(x)\ dx + \int_5^7 f(x)\ dx &\Leftrightarrow \int_5^7 f(x)\ dx = \int_0^7 f(x)\ dx - \int_0^5 f(x)\ dx \\ &\Leftrightarrow \int_5^7 f(x)\ dx = -10 - 3 \\ &\Leftrightarrow \int_5^7 f(x)\ dx = \boxed{-13}\end{aligned}$$

   (c)

$$\begin{aligned}\int_7^0 f(x)\ dx &= -\int_0^7 f(x)\ dx \\ &= -(-10) \\ &= \boxed{10}\end{aligned}$$

□

4. Evaluate $\displaystyle\int_0^{10} (60x - 6x^2)\ dx$

   Solution

$$\begin{aligned}\int_0^{10} (60x - 6x^2)\ dx &= (30x^2 - 2x^3)\Big|_0^{10} \\ &= (30 \cdot 10^2 - 2 \cdot 10^3) - (30 \cdot 0^2 - 2 \cdot 0^3) \\ &= (3000 - 2000) - 0 \\ &= \boxed{1000}\end{aligned}$$

□

5. Evaluate $\int_4^{16} 3\sqrt{x}\ dx$

Solution

$$\begin{aligned}\int_4^{16} 3\sqrt{x}\ dx &= 3\int_4^{16} x^{1/2}\ dx \\ &= 3\cdot\frac{2}{3}x^{3/2}\bigg|4^{16} \\ &= 2(16^{3/2}-4^{3/2}) \\ &= 2(64-8) \\ &= \boxed{112}\end{aligned}$$

□

6. Evaluate $\int_0^{2\pi} 3\sin x\ dx$

Solution

$$\begin{aligned}\int_0^{2\pi} 3\sin x\ dx &= -3\cos x|_0^{2\pi} \\ &= -3(\cos 2\pi - \cos 0) \\ &= -3(1-1) \\ &= \boxed{0}\end{aligned}$$

□

7. Evaluate $\int_{1/16}^{1/14} \frac{\sqrt{t}-1}{t}\ dt$

Solution

$$\begin{aligned}\int_{1/16}^{1/14} \frac{\sqrt{t}-1}{t}\ dt &= \int_{1/16}^{1/4}\left(\frac{1}{\sqrt{t}}-\frac{1}{t}\right)\ dt \\ &= \int_{1/16}^{1/4}\left(t^{-1/2}-\frac{1}{t}\right)\ dt \\ &= (2t^{1/2}-\ln|t|)\ |_{1/16}^{1/4} \\ &= \left(2\left(\frac{1}{4}\right)^{1/2}-\ln\frac{1}{4}\right)-\left(2\left(\frac{1}{16}\right)^{1/2}-\ln\frac{1}{16}\right) \\ &= 1-\ln\frac{1}{4}-\frac{1}{2}+\ln\frac{1}{16} \\ &= \boxed{\frac{1}{2}-\ln 4}\end{aligned}$$

□