ACMAT161 Summer 2024
Professor Manguba-Glover
Classwork 23   Name: _____

Complete as many of the following problems as you can with your table in the allotted time. You do not have to go in order.

1. Evaluate $\int 2(2x+1)^3 \, dx$

   Solution   Let $u = 2x + 1 \Rightarrow du = 2 \, dx$

   $$\int 2(2x+1)^3 \, dx = \int u^3 \, du$$

   $$= \frac{u^4}{4} + C$$

   $$= \boxed{\frac{(2x+1)^4}{4} + C}$$

   $\square$

2. Evaluate $\int 10e^{10x} \, dx$

   Solution

   $$u = 10x \Rightarrow du = 10 \, dx$$

   $$\int 10e^{10x} \, dx = \int e^u \, du$$
   $$= e^u + C$$
   $$= \boxed{e^{10x} + C}$$

   $\square$

3. Evaluate $\int x^4(x^5+6)^9 \, dx$

   Solution

   $$u = x^5 + 6 \Rightarrow du = 5x^4 \, dx$$

   $$\int x^4(x^5+6)^9 \, dx = \frac{1}{5} \int u^9 \, du$$

   $$= \frac{1}{5} \cdot \frac{u^{10}}{10} + C$$

   $$= \boxed{\frac{1}{50}(x^5+6)^{10} + C}$$

   $\square$

4. Evaluate $\displaystyle\int \cos^3 x \sin x \ dx$

   Solution

   $$u = \cos x \Rightarrow du = -\sin x \ dx$$

   $$\int \cos^3 x \sin x \ dx = -\int u^3 \ du$$

   $$= -\frac{u^4}{4} + C$$

   $$\boxed{= -\frac{\cos^4 x}{4} + C}$$

   $\square$

5. Evaluate $\displaystyle\int \frac{x}{\sqrt{x+1}} \ dx$

   Solution

   $$u = x + 1 \Rightarrow du = dx$$

   $$\int \frac{x}{\sqrt{x+1}} \ dx = \int \frac{u-1}{\sqrt{u}} \ du$$

   $$= \int \left(\sqrt{u} - \frac{1}{\sqrt{u}} \right) du$$

   $$= \int \left(u^{1/2} - u^{-1/2} \right) du$$

   $$= \frac{2}{3} u^{3/2} - 2u^{1/2} + C$$

   $$\boxed{= \frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C}$$

   $\square$

6. Evaluate $\displaystyle\int_0^2 \frac{dx}{(x+3)^3}$

Solution

$$u = x + 3 \Rightarrow du = dx$$

$$\int_0^2 \frac{dx}{(x+3)^3} = \int_3^5 u^{-3}\, du$$

$$= -\frac{u^{-2}}{2}\bigg|_3^5$$

$$= -\frac{1}{2}\left(5^{-2} - 3^{-2}\right)$$

$$= \frac{8}{225}$$

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7. Evaluate $\displaystyle\int_2^3 \frac{x^2}{x^3 - 7}\, dx$

Solution

$$u = x^3 - 7 \Rightarrow du = 3x^2\, dx$$

$$\int_2^3 \frac{x^2}{x^3 - 7}\, dx = \frac{1}{3}\int_1^{20} \frac{1}{u}\, du$$

$$= \frac{1}{3}\ln|u|\bigg|_1^{20}$$

$$= \frac{1}{3}\left(\ln 20 - \ln 1\right)$$

$$= \boxed{\frac{1}{3}\ln 20}$$

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8. Evaluate $\displaystyle\int_0^{\pi/2} \sin^4 x \cos x \; dx$

Solution

$$u = \sin x \Rightarrow du = \cos x \; dx$$

$$\int_0^{\pi/2} \sin^4 x \cos x \; dx = \int_0^1 u^4 \; du$$

$$= \left. \frac{u^5}{5} \right|_0^1$$

$$= \boxed{\frac{1}{5}}$$

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