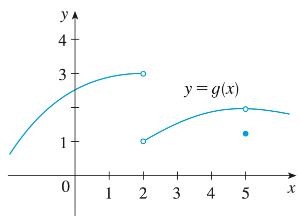
ACMAT161 Summer 2024
Professor Manguba-Glover
Classwork 3 & 4 Name: ______________________________

Complete as many of the following problems as you can with your table in the allotted time. You do not have to go in order.

Classwork 3

1. Using the graph of $g(x)$ below find the following. Approximate as needed.



(a) $\lim_{x\to 2^-} g(x)$

(b) $\lim_{x\to 2^+} g(x)$

(c) $\lim_{x\to 2} g(x)$

(d) $f(2)$

(e) $\lim_{x\to 5^-} g(x)$

(f) $\lim_{x\to 5^+} g(x)$

(g) $\lim_{x\to 5} g(x)$

(h) $g(5)$

(i) $\lim_{x\to 5} \sqrt{g(x)+7}$

Solution

(a) 3

(b) 1

(c) $\lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x)$, so the limit $\boxed{\text{DNE}}$

(d) Undefined

(e) 2

(f) 2

(g) 2

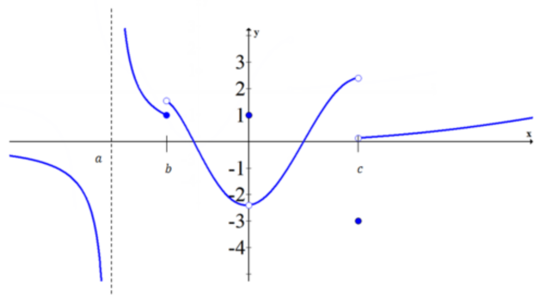
(h) ≈ 1.25

(i)

$$\begin{aligned}\lim_{x\to 5} \sqrt{g(x)+7} &= \sqrt{\lim_{x\to 5} g(x)+7} \\ &= \sqrt{2+7} \\ &= \sqrt{9} \\ &= \boxed{3}\end{aligned}$$

□

2. Using the graph of $f(x)$ below find the following. Approximate as needed.



(a) $\lim_{x \to -\infty} f(x)$

(b) $\lim_{x \to 0} f(x)$

(c) $\lim_{x \to a^-} f(x)$

(d) $\lim_{x \to a^+} f(x)$

(e) $\lim_{x \to a} f(x)$

(f) $\lim_{x \to b^-} f(x)$

Solution

(a) 0

(b) ≈ -2.4

(c) $-\infty$

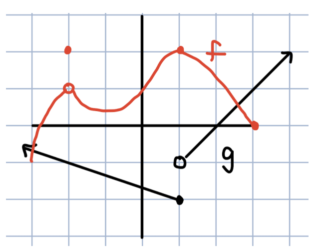
(d) ∞

(e) $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, so the limit $\boxed{\text{DNE}}$

(f) 1

□

3. Use the graph below (assuming all tick marks are 1 unit) to answer the following.



(a) $\lim_{x \to -2} (f(x) + 5g(x))$

(b) $\lim_{x \to 3} \dfrac{f(x)}{g(x)}$

Solution

(a)

$$\begin{aligned} \lim_{x \to -2} (f(x) + 5g(x)) &= \lim_{x \to -2} + 5 \lim_{x \to -2} g(x) \\ &= 1 + 5(-1) \\ &= 1 - 5 \\ &= \boxed{-4} \end{aligned}$$

(b)

$$\begin{aligned} \lim_{x \to 3} \frac{f(x)}{g(x)} &= \frac{\lim_{x \to 3}}{\lim_{x \to 3} g(x)} \\ &= \frac{0}{1} \\ &= \boxed{0} \end{aligned}$$

□

Classwork 4

1. Evaluate $\lim_{x\to 5}(2x^2 - 3x + 4)$

   Solution

$$\begin{aligned}\lim_{x\to 5}(2x^2 - 3x + 4) &= 2(5)^2 - 3(5) + 4\\ &= 2(25) - 15 + 4\\ &= 50 - 15 + 4\\ &= \boxed{39}\end{aligned}$$

□

2. Evaluate $\lim_{x\to -2}\dfrac{x^3 + 2x^2 - 1}{5 - 3x}$

   Solution

$$\begin{aligned}\lim_{x\to -2}\frac{x^3 + 2x^2 - 1}{5 - 3x} &= \frac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)}\\ &= \frac{-8 + 2(4) - 1}{5 + 6}\\ &= \frac{-8 + 8 - 1}{5 + 6}\\ &= \boxed{\frac{-1}{11}}\end{aligned}$$

□

3. Evaluate $\lim_{x\to 1}\dfrac{x^2 - 1}{x - 1}$

   Solution

$$\lim_{x\to 1}\frac{x^2 - 1}{x - 1} \to \frac{1 - 1}{1 - 1} \to \frac{0}{0} \text{ so we have to do More Work (MW)!}$$

$$\begin{aligned}\lim_{x\to 1}\frac{x^2 - 1}{x - 1} &= \lim_{x\to 1}\frac{\cancel{(x - 1)}(x + 1)}{\cancel{x - 1}}\\ &= \lim_{x\to 1}(x + 1)\\ &= 1 + 1\\ &= \boxed{2}\end{aligned}$$

□

4. Evaluate $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

Solution

$$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} \to \frac{\sqrt{4}-2}{4-4} \to \frac{2-2}{4-4} \to \frac{0}{0} \text{ MW!}$$

$$\begin{aligned}
\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} &= \lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} \\
&= \lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} \\
&= \lim_{x\to 4} \frac{1}{\sqrt{x}+2} \\
&= \frac{1}{\sqrt{4}+2} \\
&= \frac{1}{2+2} \\
&= \boxed{\frac{1}{4}}
\end{aligned}$$

□

5. Evaluate $\lim_{h\to 0} \frac{(3+h)^2-9}{h}$

Solution

$$\begin{aligned}
\lim_{h\to 0} \frac{(3+h)^2-9}{h} &= \lim_{h\to 0} \frac{(3+h)(3+h)-9}{h} \\
&= \lim_{h\to 0} \frac{\cancel{9}+6h+h^2-\cancel{9}}{h} \\
&= \lim_{h\to 0} \frac{6h+h^2}{h} \\
&= \lim_{h\to 0} \frac{\cancel{h}(6+h)}{\cancel{h}} \\
&= \lim_{h\to 0}(6+h) \\
&= 6+0 \\
&= \boxed{6}
\end{aligned}$$

□

6. If $f(x) = \begin{cases} x-1 & \text{if } x < 1 \\ 0 & \text{if } 1 \le x \le 4 \\ x-2 & \text{if } x > 4 \end{cases}$

Find $\lim_{x \to 1} f(x)$ and $\lim_{x \to 4} f(x)$

Solution Since $x = 1$ and $x = 4$ are where the piecewise function breaks, we have to check the left and the right limits at those points.

$$\begin{aligned} \lim_{x \to 1} f(x) &= \lim_{x \to 1^-} f(x) \\ &= \lim_{x \to 1^-} (x-1) \\ &= 1-1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} \lim_{x \to 1^+} f(x) &= \lim_{x \to 1^+} 0 \\ &= 0 \end{aligned}$$

Since the left and the right limits both equal 0, $\lim_{x \to 1} f(x) = \boxed{0}$

$$\begin{aligned} \lim_{x \to 4^-} f(x) &= \lim_{x \to 4^-} 0 \\ &= 0 \end{aligned}$$

$$\begin{aligned} \lim_{x \to 4^+} f(x) &= \lim_{x \to 4^+} (x-2) \\ &= 4-2 \\ &= 2 \end{aligned}$$

Since the left and the right limits don't equal each other, $\lim_{x \to 4} f(x)$ $\boxed{\text{DNE}}$

$\square$

7. If $g(x) = \begin{cases} x+1 & \text{if } x \neq 1 \\ \pi & \text{if } x = 1 \end{cases}$

Find $\lim_{x \to 1} g(x)$

Solution Remember that limits do not care about the function at that value, so we only need to pay attention to the $x+1$ piece.

$$\begin{aligned} \lim_{x \to 1} g(x) &= \lim_{x \to} 1(x+1) \\ &= 1+1 \\ &= \boxed{2} \end{aligned}$$

□

8. If $2x \leq g(x) \leq x^4 - x^2 + 2$, evaluate $\lim_{x \to 1} g(x)$

Solution The fact that there are inequalities present indicates that we should try to use the Squeeze Theorem.

$$\begin{aligned} \lim_{x \to 1} 2x &= 2(1) \\ &= 2 \end{aligned}$$

$$\begin{aligned} &\lim_{x \to 1} (x^4 - x^2 + 2) \\ &= (1)^4 - (1)^2 + 2 \\ &= 1 - 1 + 2 \\ &= 2 \end{aligned}$$

Because $2x \leq g(x) \leq x^4 - x^2 + 2$ and both $\lim_{x \to 1} 2x = \lim_{x \to 1} (x^4 - x^2 + 2) = 2$, then $\lim_{x \to 1} g(x) = \boxed{2}$ by the Squeeze Theorem.

□