ACMAT161 Summer 2024
Professor Manguba-Glover
Classwork 5 & 6 Name: ______________________________

Complete as many of the following problems as you can with your table in the allotted time. You do not have to go in order.

Classwork 5

1. Find $\lim\limits_{x\to -5^-} \dfrac{3x}{2x+10}$ and $\lim\limits_{x\to -5^+} \dfrac{3x}{2x+10}$. What does this say about $\lim\limits_{x\to -5} \dfrac{3x}{2x+10}$?

   Solution

$$\begin{aligned} \lim x \to -5^- \frac{3x}{2x+1} &\to \frac{3(-5)}{2(-5^-)+10} \\ &\to \frac{-15}{0^-} \\ &\to \infty \end{aligned}$$

$$\begin{aligned} \lim x \to -5^+ \frac{3x}{2x+1} &\to \frac{3(-5)}{2(-5^+)+10} \\ &\to \frac{-15}{0^+} \\ &\to -\infty \end{aligned}$$

   The limits do not match up, so $\lim\limits_{x\to -5} \dfrac{3x}{2x+10}$ $\boxed{\text{DNE}}$

   □

2. Find $\lim\limits_{x\to 5^+} \dfrac{x+1}{x-5}$

   Solution

$$\begin{aligned} \lim_{x\to 5^+} \frac{x+1}{x-5} &\to \frac{5+1}{5^+-5} \\ &\to \frac{6}{0^+} \\ &\to \boxed{\infty} \end{aligned}$$

   □

3. Find $\lim_{x \to 1} \frac{x-2}{(x-1)^2}$

Solution

$$\lim_{x \to 1} \frac{x-2}{(x-1)^2} \to \frac{1-2}{(1-1)^2}$$
$$\to \frac{-1}{0^+}$$
$$\to \boxed{-\infty}$$

□

4. Find $\lim \frac{1}{x^2-4}$ as $x \to 2$ and $x \to -2$

Solution

$$\lim_{x \to 2^-} \frac{1}{x^2-4} \to \frac{1}{(2^-)^2-4}$$
$$\to \frac{1}{0^-}$$
$$\to -\infty$$

$$\lim_{x \to 2^+} \frac{1}{x^2-4} \to \frac{1}{(2^+)^2-4}$$
$$\to \frac{1}{0^+}$$
$$\to \infty$$

$$\lim_{x \to -2^-} \frac{1}{x^2-4} \to \frac{1}{(-2^-)^2-4}$$
$$\to \frac{1}{0^+}$$
$$\to \infty$$

$$\lim_{x \to -2^+} \frac{1}{x^2-4} \to \frac{1}{(-2^+)^2-4}$$
$$\to \frac{1}{0^-}$$
$$\to -\infty$$

The limits do not match up, so $\lim_{x \to 2} \frac{1}{x^2-4}$ and $\lim_{x \to -2} \frac{1}{x^2-4}$ $\boxed{\text{DNE}}$

□

5. Find the vertical asymptotes for $y = \frac{2x^2}{x^2-1}$ then determine the functions behavior around them.

Solution

$$\frac{2x^2}{x^2-1} = \frac{2x^2}{(x-1)(x+1)}$$

The vertical asymptotes occur when the function is undefined, i.e. where you divide by zero.

$$(x-1)(x+1) = 0 \Leftrightarrow x-1 = 0 \text{ or } x+1 = 0 \Leftrightarrow \boxed{x = 1 \text{ or } x = -1}$$

$$\begin{aligned}\lim_{x\to 1^-} \frac{2x^2}{x^2-1} &\to \frac{2(1)^2}{(1^-)^2-1} \\ &\to \frac{2}{0^-} \\ &\to -\infty\end{aligned}$$

$$\begin{aligned}\lim_{x\to 1^+} \frac{2x^2}{x^2-1} &\to \frac{2(1)^2}{(1^+)^2-1} \\ &\to \frac{2}{0^+} \\ &\to \infty\end{aligned}$$

$$\begin{aligned}\lim_{x\to -1^-} \frac{2x^2}{x^2-1} &\to \frac{2(-1)^2}{(-1^-)^2-1} \\ &\to \frac{2}{0^+} \\ &\to \infty\end{aligned}$$

$$\begin{aligned}\lim_{x\to -1^-} \frac{2x^2}{x^2-1} &\to \frac{2(-1)^2}{(-1^+)^2-1} \\ &\to \frac{2}{0^-} \\ &\to -\infty\end{aligned}$$

□

6. Find the vertical asymptotes of $\frac{x^2-3x+2}{x^3-4x}$ and determine the functions behavior around them.

Solution

$$\frac{x^2-3x+2}{x^3-4x} = \frac{(x-2)(x-1)}{x(x^2-4)} = \frac{(x-2)(x-1)}{x(x-2)(x+2)}$$

The $x-2$ will cancel, meaning that there is not a vertical asymptote there, there is just a hole in the graph at $x=2$. The other factors in the denominator will produce vertical asymptotes, so the asymptotes are $\boxed{x=0 \text{ and } x=-2}$.

$$\begin{aligned}\lim_{x\to 0^-}\frac{(x-2)(x-1)}{x(x-2)(x-1)} &= \lim_{x\to 0^-}\frac{x-1}{x(x+2)}\\ &\to \frac{0-1}{0^-(0+2)}\\ &\to \frac{-1}{0^-(2)}\\ &\to \infty\end{aligned}$$

$$\begin{aligned}\lim_{x\to 0^+}\frac{(x-2)(x-1)}{x(x-2)(x-1)} &= \lim_{x\to 0^+}\frac{x-1}{x(x+2)}\\ &\to \frac{0-1}{0^+(0+2)}\\ &\to \frac{-1}{0^+(2)}\\ &\to -\infty\end{aligned}$$

$$\begin{aligned}\lim_{x\to -2^-}\frac{(x-2)(x-1)}{x(x-2)(x-1)} &= \lim_{x\to -2^-}\frac{x-1}{x(x+2)}\\ &\to \frac{-2-1}{-2(-2^-+2)}\\ &\to \frac{-3}{-2(0^-)}\\ &\to -\infty\end{aligned}$$

$$\begin{aligned}\lim_{x\to -2^+}\frac{(x-2)(x-1)}{x(x-2)(x-1)} &= \lim_{x\to -2^+}\frac{x-1}{x(x+2)}\\ &\to \frac{-2-1}{-2(-2^++2)}\\ &\to \frac{-3}{-2(0^+)}\\ &\to \infty\end{aligned}$$

□

Classwork 6

1. Find $\lim_{x\to\infty}(x^2 - x)$

   Solution

$$\begin{aligned}\lim_{x\to\infty}(x^2 - x) &= \lim_{x\to\infty} x^2\\ &\to (\infty)^2\\ &\to \boxed{\infty}\end{aligned}$$

□

2. Evaluate $\lim_{x\to\infty}\frac{3x^2 - x - 2}{5x^2 + 4x + 1}$

   Solution

$$\begin{aligned}\lim_{x\to\infty}\frac{3x^2 - x - 2}{5x^2 + 4x + 1} &= \lim_{x\to\infty}\frac{\frac{3x^2}{x^2} - \frac{x}{x^2} - \frac{2}{x^2}}{\frac{5x^2}{x^2} + \frac{4x}{x^2} + \frac{1}{x^2}}\\ &= \lim_{x\to\infty}\frac{3 - \frac{1}{x} - \frac{2}{x^2}}{5 + \frac{4}{x} + \frac{1}{x^2}}\\ &= \frac{3 - 0 - 0}{5 + 0 + 0}\\ &= \boxed{\frac{3}{5}}\end{aligned}$$

□

3. Find $\lim_{x\to\infty}\frac{x^2 + 8}{6x^2 - x}$

   Solution

$$\begin{aligned}\lim_{x\to\infty}\frac{x^2 + 8}{6x^2 - x} &= \lim_{x\to\infty}\frac{\frac{x^2}{x^2} + \frac{8}{x^2}}{\frac{6x^2}{x^2} - \frac{x}{x^2}}\\ &= \lim_{x\to\infty}\frac{1 + \frac{8}{x^2}}{6 - \frac{1}{x}}\\ &= \frac{1 + 0}{6 - 0}\\ &= \boxed{\frac{1}{6}}\end{aligned}$$

□

4. Find $\lim_{x \to \infty} \frac{\pi\sqrt{2}}{x^3}$

Solution

$$\begin{aligned}\lim_{x \to \infty} \frac{\pi\sqrt{2}}{x^3} &\to \frac{\pi\sqrt{2}}{\infty^3} \\ &\to \frac{\pi\sqrt{2}}{\infty} \\ &\to \boxed{0}\end{aligned}$$

□

5. Find $\lim_{x \to \infty} \frac{x+5}{x^3+7x^2+1}$

Solution

$$\begin{aligned}\lim_{x \to \infty} \frac{x+5}{x^3+7x^2+1} &= \lim_{x \to \infty} \frac{\frac{x}{x^3}+\frac{5}{x^3}}{\frac{x^3}{x^3}+\frac{7x^2}{x^3}+\frac{1}{x^3}} \\ &= \lim_{x \to \infty} \frac{\frac{1}{x^2}+\frac{5}{x^3}}{1+\frac{7}{x}+\frac{1}{x^3}} \\ &= \frac{0+0}{1+0+0} \\ &= \boxed{0}\end{aligned}$$

□

6. Find $\lim_{x \to -\infty} \frac{x^4+3x^3+x-1}{x+1}$

Solution

$$\begin{aligned}\lim_{x \to -\infty} \frac{x^4+3x^3+x-1}{x+1} &= \lim_{x \to -\infty} \frac{\frac{x^4}{x}+\frac{3x^3}{x}+\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}+\frac{1}{x}} \\ &= \lim_{x \to -\infty} \frac{x^3+3x^2+1-\frac{1}{x}}{1+\frac{1}{x}} \\ &\to (-\infty)^3 \\ &\to \boxed{-\infty}\end{aligned}$$

□

7. Find the end behavior of $f(x) = \frac{x^2-1}{x^2+1}$ to determine the horizontal asymptote(s)

Solution

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^2-1}{x^2+1} &= \lim_{x\to\infty} \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} \\
&= \lim_{x\to\infty} \frac{1 - \frac{1}{x^2}}{1 + \frac{1}{x^2}} \\
&= \frac{1-0}{1+0} \\
&= 1
\end{aligned}$$

$$\begin{aligned}
\lim_{x\to-\infty} \frac{x^2-1}{x^2+1} &= \lim_{x\to-\infty} \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} \\
&= \lim_{x\to-\infty} \frac{1 - \frac{1}{x^2}}{1 + \frac{1}{x^2}} \\
&= \frac{1-0}{1+0} \\
&= 1
\end{aligned}$$

This gives us that the horizontal asymptote is $\boxed{y = 1}$

□

8. Determine if there is an asymptote for $f(x) = \frac{x^2+x}{3-x}$

Solution

$$\frac{x^2+x}{3-x} = \frac{x^2+x}{-x+3}$$

The degree of the denominator is one fewer than the degree of the numerator, so there is a slant asymptote. We can do long division to find it.

$$\begin{array}{r|l}
 & -x \quad -4 \\
\hline
-x+3 & x^2 \quad +x \\
 & -x^2 + 3x \\
\hline
 & 4x \\
 & -4x + 12 \\
\hline
 & 12
\end{array}$$

This gives us that the slant asymptote is $\boxed{y = -x - 4}$

□

9. Determine the asymptotes of $f(x) = \frac{x^3}{x^2+1}$

Solution The degree of the denominator is one fewer than the degree of the numerator, so there is a slant asymptote. We can do long division to find it.

$$\begin{array}{r l} & \phantom{-\,x^3 - {}} x \\ \cline{2-2} x^2+1 \big) & \;\; x^3 \\ & -\,x^3 - x \\ \cline{2-2} & - x \end{array}$$

This gives us that the slant asymptote is $\boxed{y = x}$

$\square$