Midterm 1 Practice Questions

Note: This document is split up based on major mathematical themes covered so far. A topic that is not on this practice exam may still show up on the actual exam if it was covered in class.

General Limits

(1) Evaluate $\lim_{x\to -1} \frac{x^2-5x-6}{x+1}$

(2) $\lim_{x\to 25} \frac{x-25}{\sqrt{x}-5}$

(3) Evaluate $\lim_{t\to 1} \frac{t^2+t-2}{t^2-1}$

(4) Evaluate $\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2}$

(5) Evaluate $\lim_{x\to 0} \frac{1}{x^3-1}+1$

(6) Evaluate $\lim_{x\to 1} \frac{3x-4}{x^2+x+1}$

(7) Evaluate $\lim_{x\to -1^-} \frac{|x+1|}{x+1}$

(8) Evaluate $\lim_{x\to 1} \frac{x^2-1}{x-1}$

(9) Evaluate $\lim_{x\to 0} x\cos\left(\frac{1}{x}\right)$

(10) Evaluate $\lim_{x\to 3^-} \frac{|x-3|}{(6-2x)}$

(11) Evaluate $\lim_{h\to 1} \frac{\sqrt{h+8}-3}{1-h}$

(12) Evaluate $\lim_{x\to 0} x^2\cos\left(\frac{3}{x}\right)-11$

(13) Evaluate the following limits for $f(x)$

$$f(x) = \begin{cases} x^2-3x+4 & x \le 1 \\ x+1 & 1 < x \le 3 \\ x^2-3x+4 & x > 3 \end{cases}$$

(a) $\lim_{x\to 1^-} f(x)$

(b) $\lim_{x\to 1^+} f(x)$

(c) $\lim_{x\to 1} f(x)$

(d) $\lim_{x\to 3^-} f(x)$

(e) $\lim_{x\to 3^+} f(x)$

(f) $\lim_{x\to 3} f(x)$

Infinite Limits & Limits at Infinity

(1) Evaluate $\lim_{x\to\infty} \frac{2x^3 - 4x^2 + 5x}{17x^2 + 1}$

(2) Evaluate $\lim_{x\to -1} \frac{x^2 + x - 2}{x^3 + 1}$

(3) $\lim_{x\to\infty} \frac{x^3 + x^2 + 2}{7x^3 + x + 1}$

(4) $\lim_{x\to 3^-} \frac{4}{(x-3)^2}$

(5) $\lim_{x\to\infty} \frac{7x^3}{x^3 - 3x^2 + 6x}$

(6) $\lim_{x\to -\infty} \frac{3x^2 + 4}{x + 7}$

(7) $\lim_{t\to 2^-} \frac{t + 2}{t - 2}$

(8) $\lim_{x\to\infty} \frac{2x^2 - 11x + 5}{3x - 7}$

Continuity

(1) Describe on which interval(s) the following function is continuous: $y = \frac{\sin x}{x-2}$

(2) Describe on which interval(s) the following function is continuous:

$$f(x) = \begin{cases} 3 - x, & x < 2 \\ \frac{x}{2} + 1, & 2 \leq x < 4 \\ 3, & x \geq 4 \end{cases}$$

(3) Describe on which interval(s) the following function is continuous (show your work):

$$f(x) = \begin{cases} 1 - x^2 & x < -1 \\ 1 + x & -1 \leq x \leq 1 \\ -3 & x > 1 \end{cases}$$

(4) Is the following function continuous:

$$f(x) = \begin{cases} x + 3 & x < 2 \\ 5 & x = 2 \\ x^4 - 11 & x > 2 \end{cases}$$

(5) What values of m and b make the following function continuous:

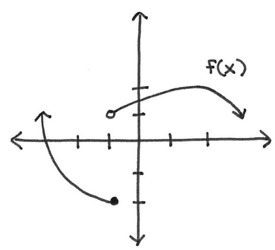
$$f(x) = \begin{cases} x^2 - 7 & x < -2 \\ mx + b & -2 \leq x \leq 2 \\ 5 & x > 2 \end{cases}$$

(6) Show that the equation $x^3 - x^2 + 2x - 7 = 0$ has a solution in the interval $[1, 2]$. State any theorems you use to support your answer.

(7) Let $f(x) = 5 + x - x^4$. Use the intermediate value theorem to show that there is at least one point where $f(x) = 0$.

Reading/Using Graphs

(1) From the picture, decide whether the following limits exist. If they exist, find their value:



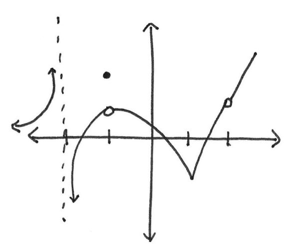
(a) $\lim_{x \to -1^-} f(x)$

(b) $\lim_{x \to -1^+} f(x)$

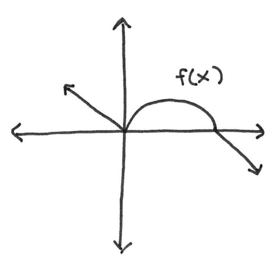
(c) $\lim_{x \to -1} f(x)$

(d) $\lim_{x \to 2} f(x)$

(2) Identify the x-values where $f(x)$ is discontinuous.



(3) Sketch the graph of the derivative of the function shown.



(4) Consider the function $f(x)$ given below. Find

(i) $\lim_{x \to k^-} f(x)$

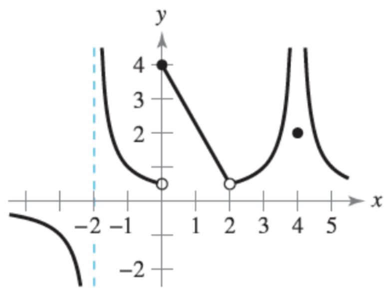
(ii) $\lim_{x \to k^+} f(x)$

(iii) $\lim_{x \to k} f(x)$

(iv) $f(k)$

(v) Is $f(x)$ continuous at k? (yes or no)

for each of the given values of k. If the given value does not exist, write "DNE", ∞, $-\infty$, or "undefined" as necessary:



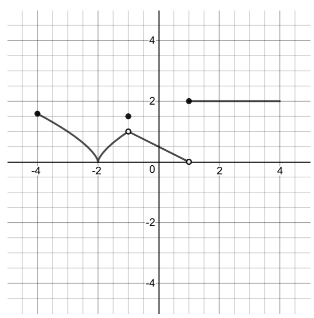
(a) $k = -1$

(b) $k = 0$

(c) $k = 2$

(d) $k = 4$

(5) The function $f(x)$ is defined for $-4 \le x \le 4$ and is graphed below. Use the graph to answer the following questions:



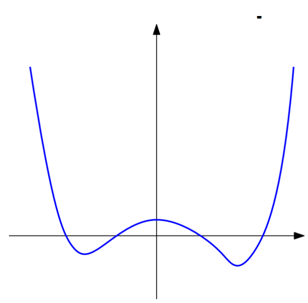
(a) What is $\lim_{x \to -1} f(x)$?

(b) What is $\lim_{x \to 1} f(x)$?

(c) Give the intervals where $f(x)$ is continuous, be careful to include the endpoints if necessary.

(d) Does the function appear to be differentiable at $x = -2$? Explain why or why not.

(e) Sketch the graph of f' given that the graph of f looks like the following:



Limit Definition of a Derivative

(1) (a) Using the limit definition of a derivative, differentiate the following:

$$f(x) = x^2 - 3x - 1$$

(b) Find the equation of the line tangent to $f(x)$ at $x = 1$

(2) (a) **Use the definition of derivative** to show that the derivative of $f(x) = x^2 - x$ at $x = -2$ is -5, i.e. $f'(-2) = -5$.

(b) Find an equation for the tangent line to $f(x) = x^2 - x$ at $x = -2$.

(3) Use the definition of the derivative (limit definition) to find the derivative of $f(x) = \sqrt{x}$

(4) Use the definition of the derivative (limit definition) to find the derivative of $f(x) = x^2 - x$

(5) Use the definition of the derivative (limit definition) to find the derivative of $f(x) = \frac{1}{x}$

(6) Consider the function $f(x) = 5 - x^2$.

(a) Find the equation for the secant line to the graph of $f(x)$ that passes through the points $(1,4)$ and $(2,1)$.

(b) Find $f'(x)$ using the definition of a derivative.

(c) Find the equation for the tangent line to the graph of $f(x)$ at the point $(1,4)$.

(d) Find the equation for the tangent line to the graph of $f(x)$ at the point $(2,1)$.

Basic Differentiation

(1) Differentiate. You do not need to simplify your answer: $y = 3x \sin x$

(2) Differentiate. You do not need to simplify your answer: $y = \frac{x+1}{x^2+2}$

(3) Differentiate. You do not need to simplify your answer: $f(x) = 12x^2 - \frac{5}{\sqrt{x}} + 78$

(4) Differentiate. You do not need to simplify your answer: $y = \frac{x^3+1}{2-x}$

(5) Differentiate. You do not need to simplify your answer: $y = 6x^2 - 10x - 5x^{-2}$

(6) Differentiate. You do not need to simplify your answer: $y = x^2 \sin x + 2x \cos x - 2 \sin x$

(7) Differentiate. You do not need to simplify your answer: $y = \frac{\cot x}{1+\cot x}$

(8) Differentiate. You do not need to simplify your answer: $f(x) = \sin x(x^2 + 3) + x^{4/3}$

(9) Differentiate. You do not need to simplify your answer: $f(x) = 2x^3 e^x + 1$

(10) Differentiate. You do not need to simplify your answer: $f(x) = x \sin^{-1} x$

(11) Differentiate. You do not need to simplify your answer: $h(x) = \frac{\log_2 x}{\cos^{-1} x}$

(12) Find an equation for the tangent line to the graph of $y = \dfrac{3}{x+1}$ for $x = 1$

(13) Find an equation of the tangent line to the curve $y = 2 - x^3$ at $(1, 1)$

Chain Rule

(1) Differentiate. You do not need to simplify your answer: $y = \frac{\cot x}{1+\cot(x^2+x)}$

(2) Differentiate. You do not need to simplify your answer: $h(x) = x\tan\left(2\sqrt{x}\right) + 7$

(3) Differentiate. You do not need to simplify your answer: $h(t) = \cos^2(\pi t) + 3$

(4) Differentiate. You do not need to simplify your answer: $y = \frac{\sin^2 x}{2x^3+4x^2+7}$

(5) Differentiate. You do not need to simplify your answer: $y = \tan\left(x^2 + 1\right)$

(6) Differentiate. You do not need to simplify your answer: $y = \left(1 - \frac{x}{7}\right)^{-7}$

(7) Differentiate. You do not need to simplify your answer: $y = 4x^2(3x-2)^5$

(8) Differentiate. You do not need to simplify your answer: $\ln\left(x^4 + 1\right)$

(9) Differentiate. You do not need to simplify your answer: $2^{3+\sin x}$

(10) Differentiate. You do not need to simplify your answer: $\log_2 \frac{8}{\sqrt{2x+1}}$

(11) Differentiate. You do not need to simplify your answer: $\tan^{-1}(e^{4x})$

Implicit Differentiation

(1) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ if $2\sqrt{y} = x - y$

(2) Find an equation of the tangent line to $x^2 + xy + y^2 = 3$ at $(1, 1)$

(3) Find $\dfrac{dy}{dx}$ if $y \sin\left(\dfrac{1}{y}\right) = 1 - xy$.

(4) Find an equation for the tangent line to $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at $(-1, 0)$.

(5) Find y' at $(1, 1)$ for $y^4 - \sqrt{x} + 2y = 2$

Related Rates

(1) The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12cm and the width is 5cm, find the rates of change of **a)** the area, **b)** the perimeter, and **c)** the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

(2) A rectangular plot of land will be bounded on one side by a river and on the other three sides by some sort of fence. With 800 m of fencing at your disposal, what is the largest area you can enclose, and what are its dimensions?

(3) A child flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from them at a rate of 25 ft/sec. How fast must they let out the string when the kite is 500 ft away from them?

(4) The volume V of a circular cylinder of height h and radius r is given by $V = \pi r^2 h$. Assume that V is kept constant ($V = 8\pi$) as r and h are changing. Calculate the rate of change of h with respect to r when $h = 2$ and $r = 2$.

(5) A spherical snowball is placed in the sun. The sun melts the snowball so that its radius **decreases** 1/4 in. per hour. Find the rate of change of the volume with respect to time at the instant the radius is 4 in. The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

(6) A person leaves a given point and travels north at 3 mph. Another person leaves the same point at the same time and travels east at 4 mph. At what rate is the distance between the two people changing at the instant when they have traveled 2 hours?