Reference Sheet

Limits:

- The Squeeze Theorem: If $f(x) \le g(x) \le h(x)$ for x near a and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$
- $\lim_{x\to\pm\infty} (a_n x^n + a_{n-a}x^{n-a} + \ldots + a_1 x + a_0) = \lim_{x\to\pm\infty} a_n x^n$
- $\lim_{x\to\infty} x^n = \infty$ and $\lim_{x\to-\infty} x^n = -\infty$, if $n > 0$ is odd
- $\lim_{x\to\pm\infty} \frac{1}{x^n} = 0$ for $n > 0$
- $\lim_{x\to\pm\infty} x^n = \infty$, if $n > 0$ is even
- $\lim_{x\to\infty} e^x = \infty$
- $\lim_{x\to-\infty} e^x = 0$
- $\lim_{x\to\infty} e^{-x} = 0$
- $\lim_{x\to-\infty} e^{-x} = \infty$
- $\lim_{x\to 0^+} \ln x = -\infty$
- $\lim_{x\to\infty} \ln x = \infty$

The Intermediate Value Theorem: Suppose f is continuous on the interval $[a,b]$ and L is a number strictly between $f(a)$ and $f(b)$. Then there exists at least one number c in (a,b) satisfying $f(x) = L$.

Derivative Formulas:

- $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ OR $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$
- $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$
- $\frac{d}{dx}(f\circ g)(x) = \frac{d}{dx}(f(g(x)) = f'(g(x))g'(x)$
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(a^x) = (\ln a)a^x$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\log_a x) = \frac{d}{dx}(\log_a |x|) = \frac{1}{(\ln a)x}$
- $\frac{d}{dx}(\ln x) = \frac{d}{dx}(\ln |x|) = \frac{1}{x}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\csc^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\csc x) = -\csc x \cot x$
- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$
- $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$
- $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$
- $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

The Mean Value Theorem: Let f be a function that satisfies the following: f is continuous on $[a,b]$ and differentiable on (a,b), then there is a number c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a} \text{ or equivalently } f'(c)(b-a) = f(b)-f(a)$$

Derivatives and Graphing

- Steps for finding absolute extrema on $[a, b]$:
  1. (1) Find all critical numbers in (a, b) and evaluate the function at those values.
  2. (2) Find $f(a)$ and $f(b)$
  3. (3) Compare

- The First Derivative Test: Suppose that c is a critical number of f.
  - If f' changes from positive to negative at $x = c$, then f has a local maximum at $x = c$
  - If f' changes from negative to positive at $x = c$, then f has a local minimum at $x = c$
  - If f' does not change signs at $x = c$, then there is neither a local max nor a local min at $x = c$

- The Second Derivative Test:
  - If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$
  - if $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$
  - If $f'(c) = 0$ and $f''(c) = 0$, then the test is inconclusive.

- Steps for Curve Sketching:
  (1) Find the x and y intercepts of the function
  (2) Check for symmetry
    (a) If $f(-x) = f(x)$, the graph has y-axis symmetry
    (b) If $f(-x) = -f(x)$, the graph has origin symmetry
  (3) Determine the domain and the location of any asymptotes
  (4) Use the first derivative to find intervals of increase, intervals of decrease, and the location of any local extrema
  (5) Use the second derivative to find intervals of concavity and the location of any inflection points
  (6) Sketch the curve using the above information

- Steps for Solving Optimization Problems:
  (1) Read the problem and draw a picture if necessary.
  (2) Determine the relevant equations. Typically there are two: the objective equation and the constraint equation.
  (3) Use the constraint equation to rewrite the objective equation in terms of one variable.
  (4) Use a derivative test to find the absolute maximum or absolute minimum

Linearization Equation: $f(x) \approx L(x) = f(a) + f'(a)(x-a)$

L'Hospital's Rule (or L'Hôpital's Rule): Suppose f and g are differentiable functions with $g'(x) \neq 0$ when $x \neq a$. If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or if $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is $\pm\infty$). This also applies if $x \to \pm\infty$, $x \to a^+$, or $x \to a^-$

Integration and Antiderivatives:

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ if $n \neq -1$
- $\int kf(x) \, dx = k \int f(x) \, dx$, where k is a constant
- $\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$
- $\int e^x \, dx = e^x + C$
- $\int e^{kx} \, dx = \frac{e^{kx}}{k} + C$
- $\int a^x \, dx = \frac{a^x}{\ln a} + C$ If $a > 0$, $a \neq 1$
- $\int a^{kx} \, dx = \frac{a^{kx}}{k \ln a} + C$ If $a > 0$, $a \neq 1$
- $\int x^{-1} \, dx = \ln|x| + C$
- $\int \sin x \, dx = -\cos x + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \sec^2 x \, dx = \tan x + C$
- $\int \csc^2 x \, dx = -\cot x + C$
- $\int \sec x \tan x \, dx = \sec x + C$
- $\int \csc x \cot x \, dx = -\csc x + C$